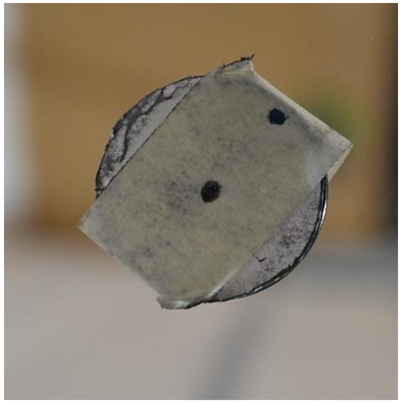
6. Ring Oiler

Dave Singh

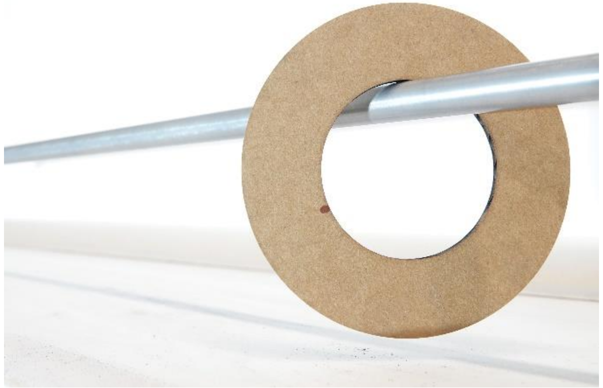
Problem Statement

"An oiled horizontal cylindrical shaft rotates around its axis at constant speed. Make a ring from a cardboard disc with the inner diameter roughly twice the diameter of the shaft and put the ring on the shaft. Depending on the tilt of the ring, it can travel along the shaft in either direction. Investigate the phenomenon."

Circular shaft with tracking dots



Ring resting on shaft with a tilt



Overview

Phenomenon

Reproduction of the Phenomenon & Experimental Measurements

Experimental Setup

Shaft-Motor setup, Camera setup, Tracker

Theoretical Model

Force balance, Torque balance and the two types of oscillations

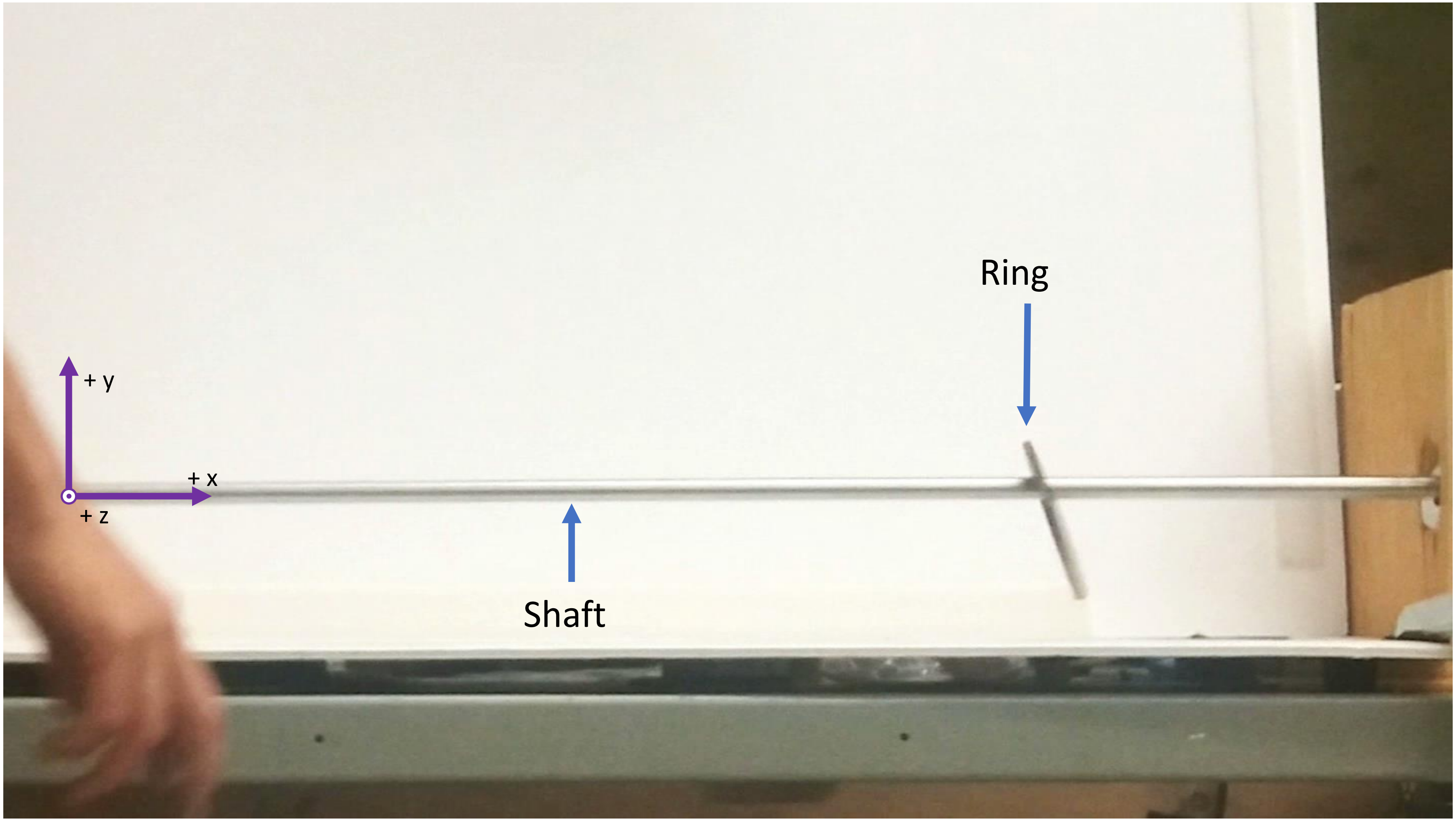
Key Parameters

Tilts relative to the z- and y-axis, Reduction in the coefficient of friction, Shaft's angular velocity, Inner radius of ring

Conclusion

General Theory and Future Improvements in the Experiment

Phenomenon



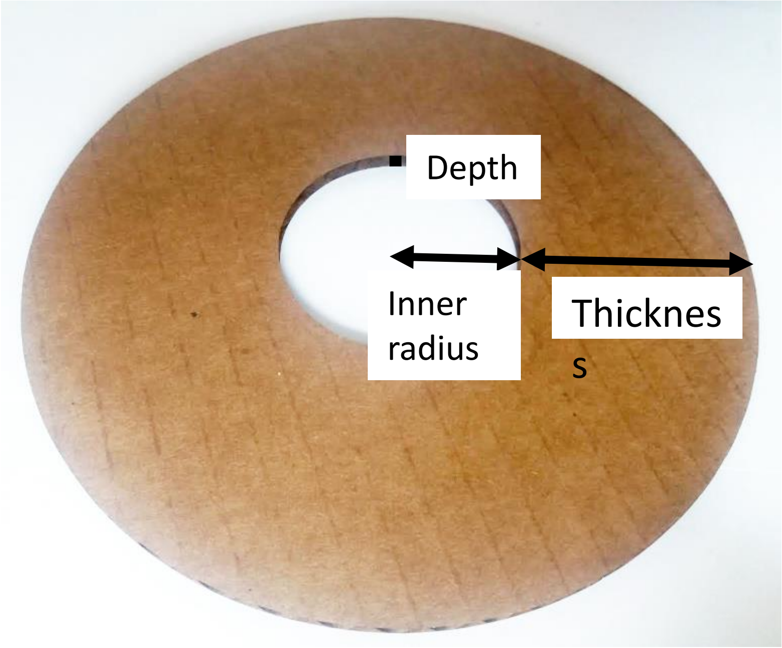
Experimental Measure 1: Ring radii, thickness and depth

Depth:
- 3.36 ± 0.01 mm

Inner radii:
- Shaft: 14.62 ± 0.01 mm
- 17.54 ± 0.01 mm
- 20.47 ± 0.01 mm
- 23.39 ± 0.01 mm
- 26.32 ± 0.01 mm
- 27.78 ± 0.01 mm
- 29.24 ± 0.01 mm
- 30.70 ± 0.01 mm
- 32.16 ± 0.01 mm

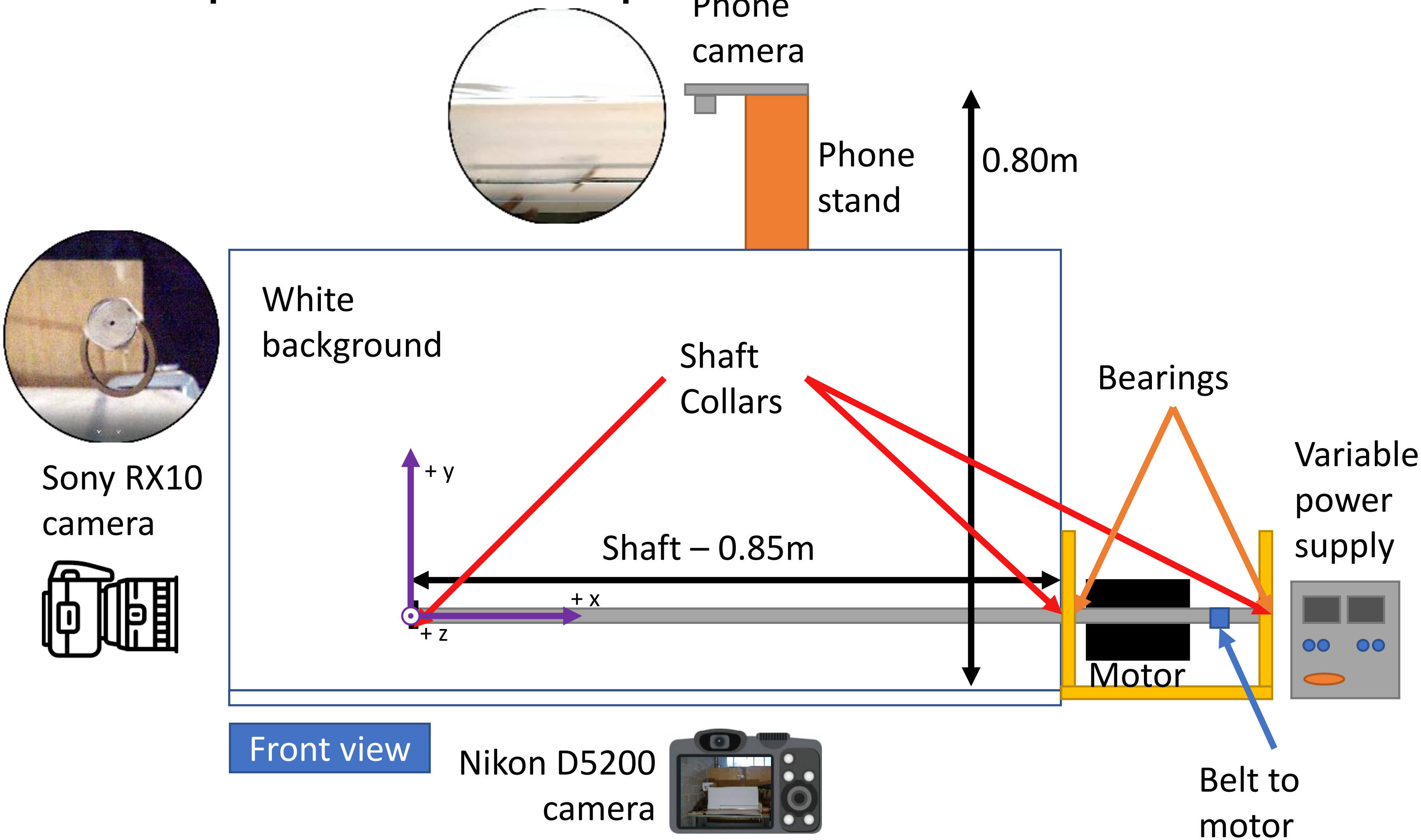
Thicknesses:
- 4.76 ± 0.01 mm
- 14.76 ± 0.01 mm
- 24.69 ± 0.01 mm

Digital Vernier Caliper: 00.00 – 200.00 ± 0.01 mm

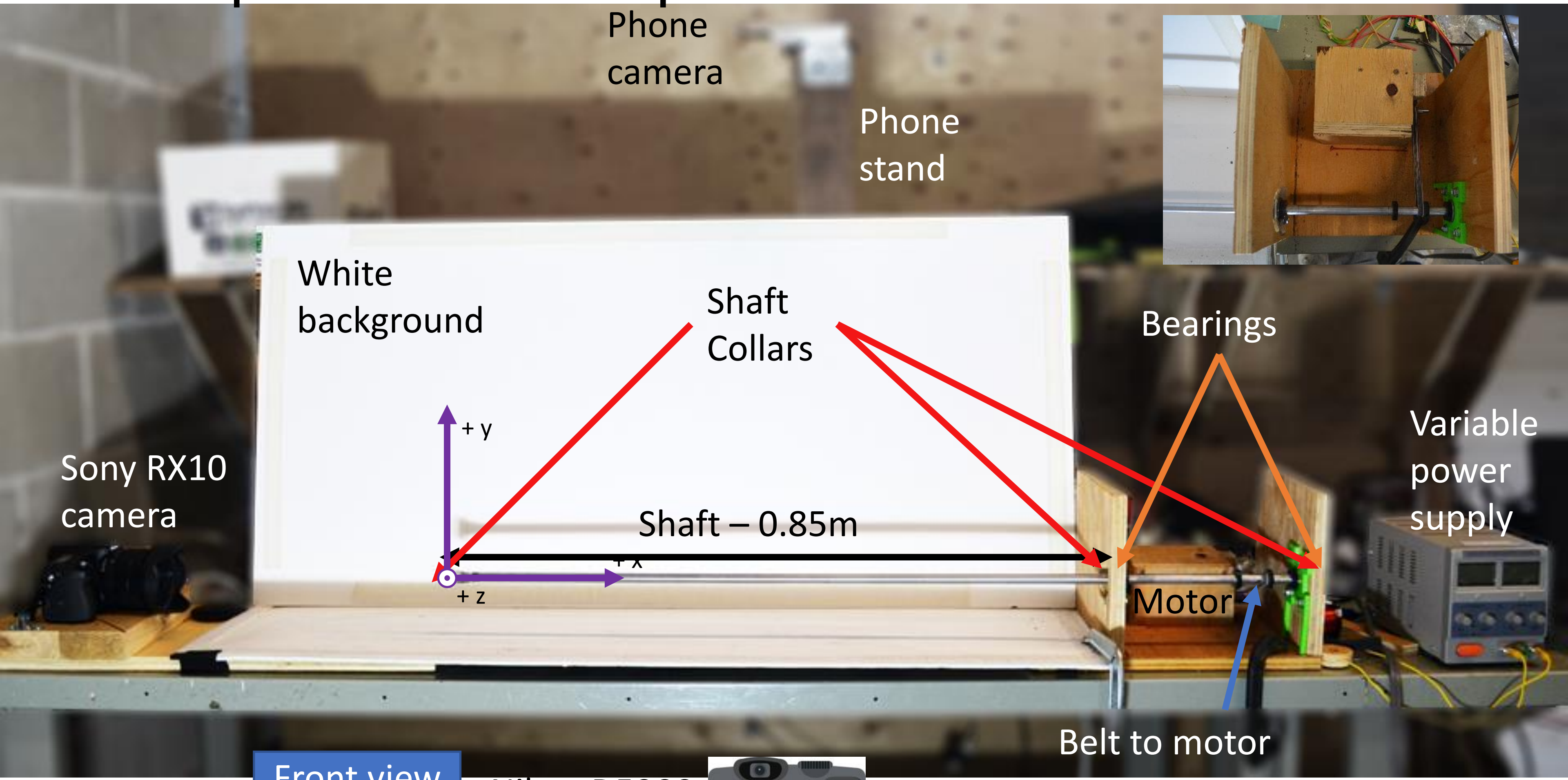


Experimental Setup

Experimental Setup



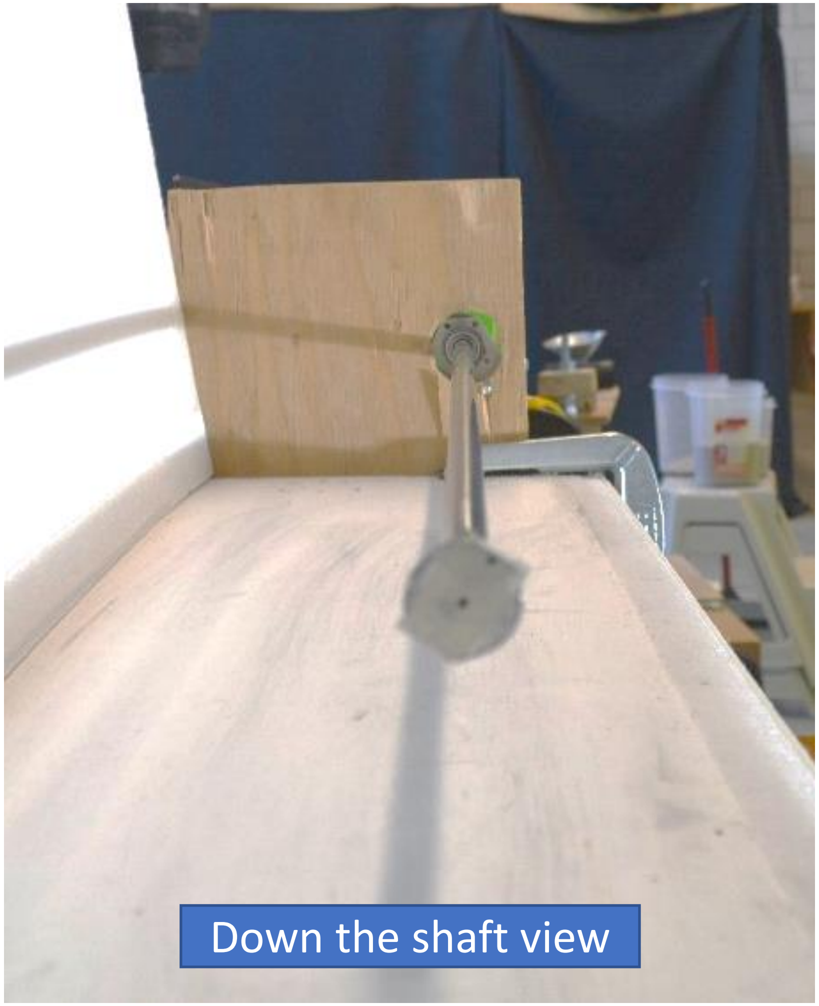
Experimental Setup

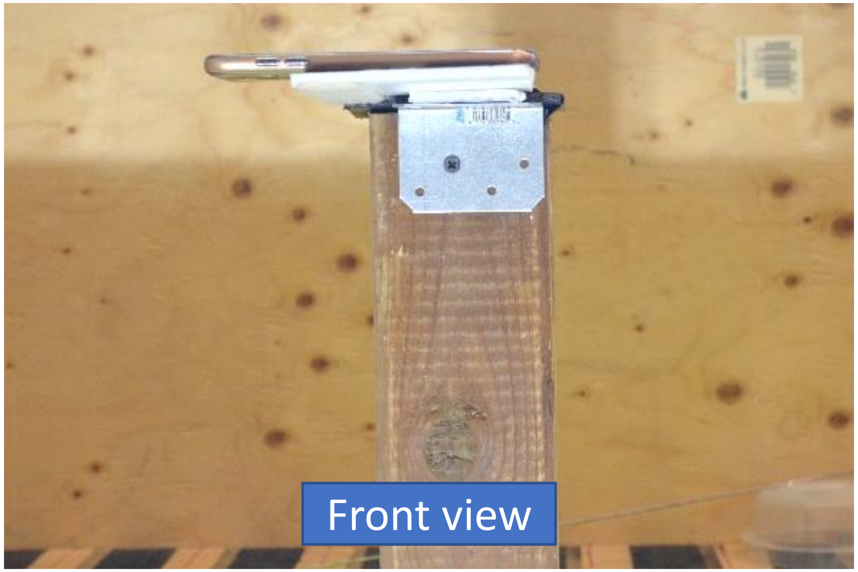


Front view

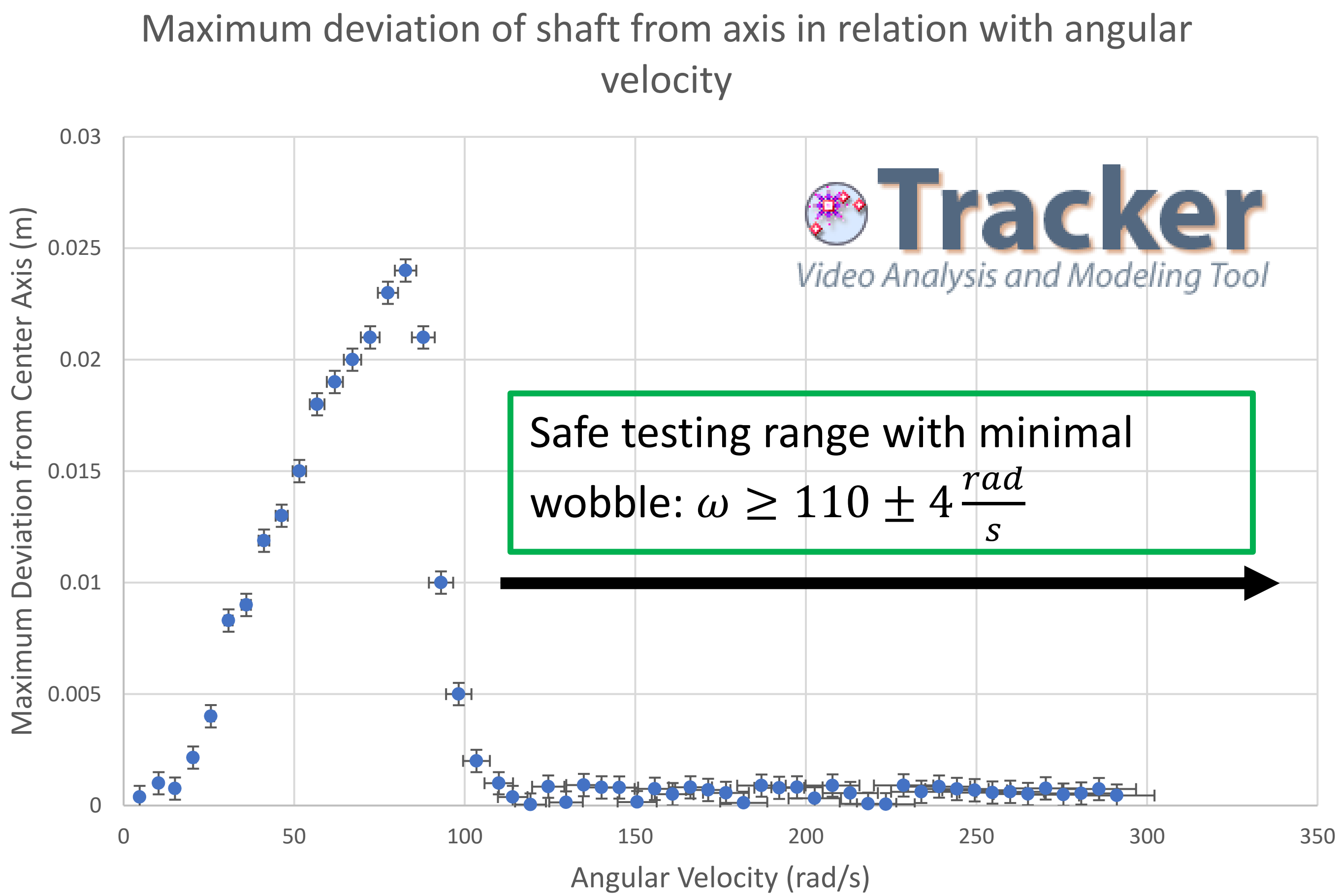
Nikon D5200 camera

Experimental Setup



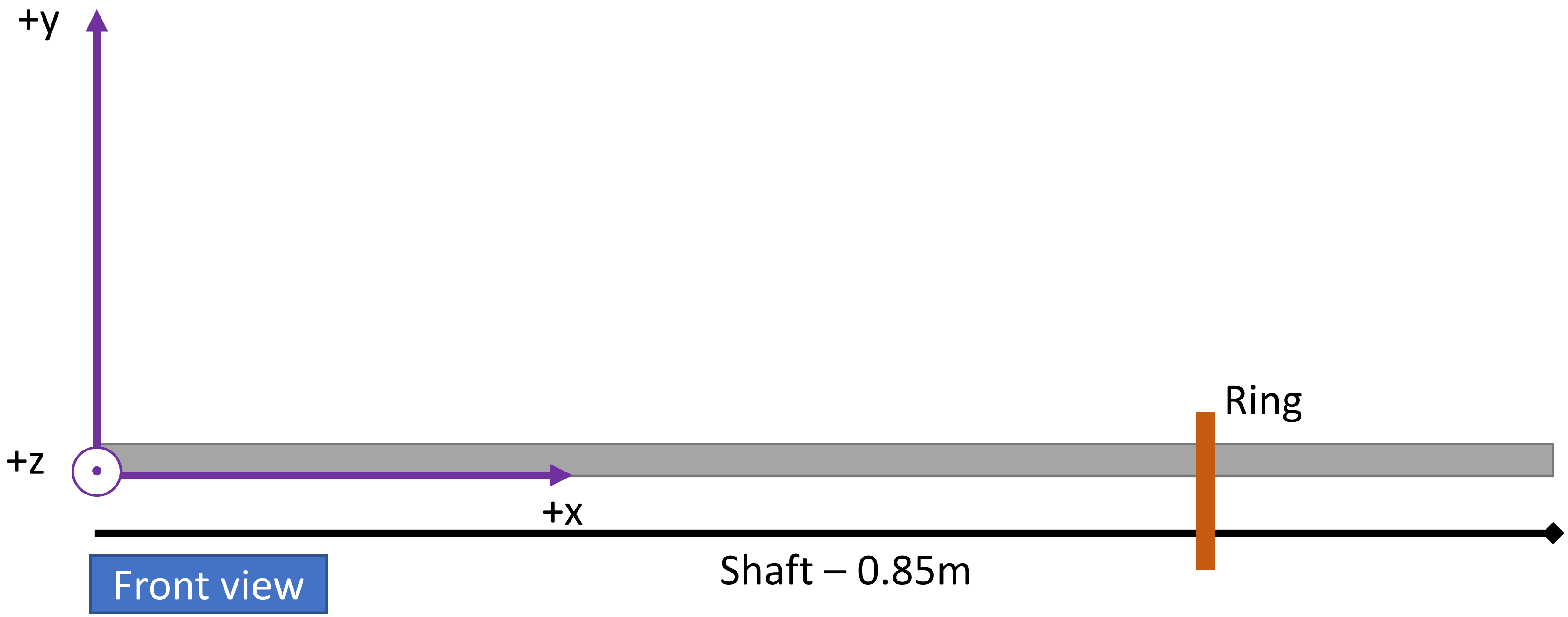


Experimental Measure 2: Shaft Wobble

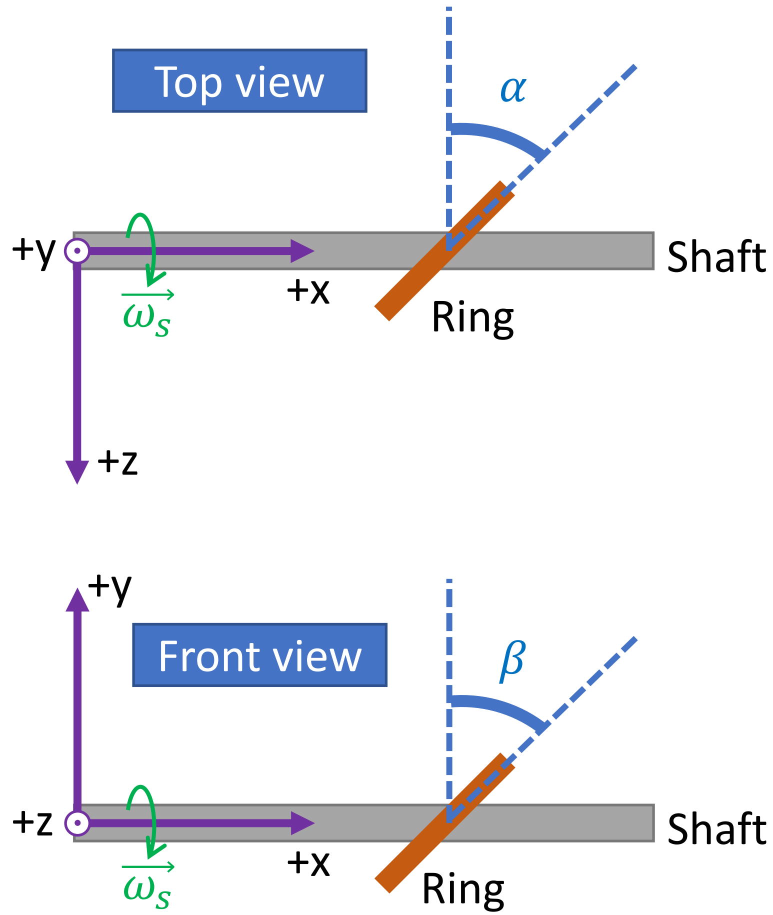


Theoretical Model

Geometry



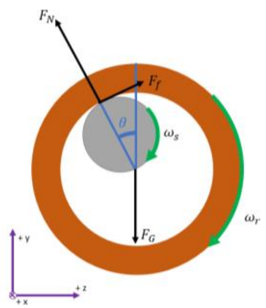
Geometry

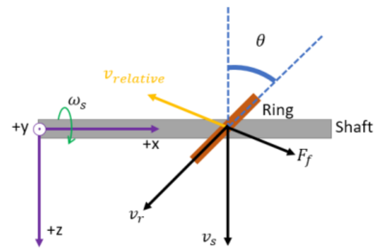

Top view
α
+y
+x
+z
$\vec{\omega_s}$
Shaft
Ring
Front view
β
+y
+z
+x
$\vec{\omega_s}$
Shaft
Ring

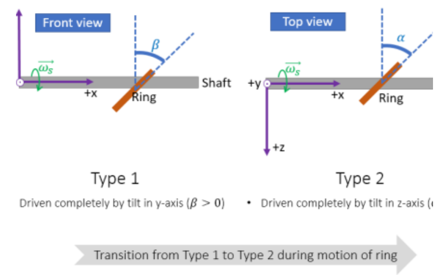
Theoretical Model Assumptions

- *Ring is a rigid body*
- *Shaft spins along only the x-axis (i.e. ring is always in contact with the shaft)*
- *Air resistance is negligible*
  - ***Verification:***
  - $F_{drag} = \frac{C_d \rho A v^2}{2}$
  - $C_d = 0.47$, $\rho = 1.225 \frac{kg}{m^3}$, $A \approx 0.04m^2$, $A \approx 0.5\ m^2$
  - $F_{drag} \approx 0.001 - 0.005$
- *Thin layer of oil*

Theoretical Model





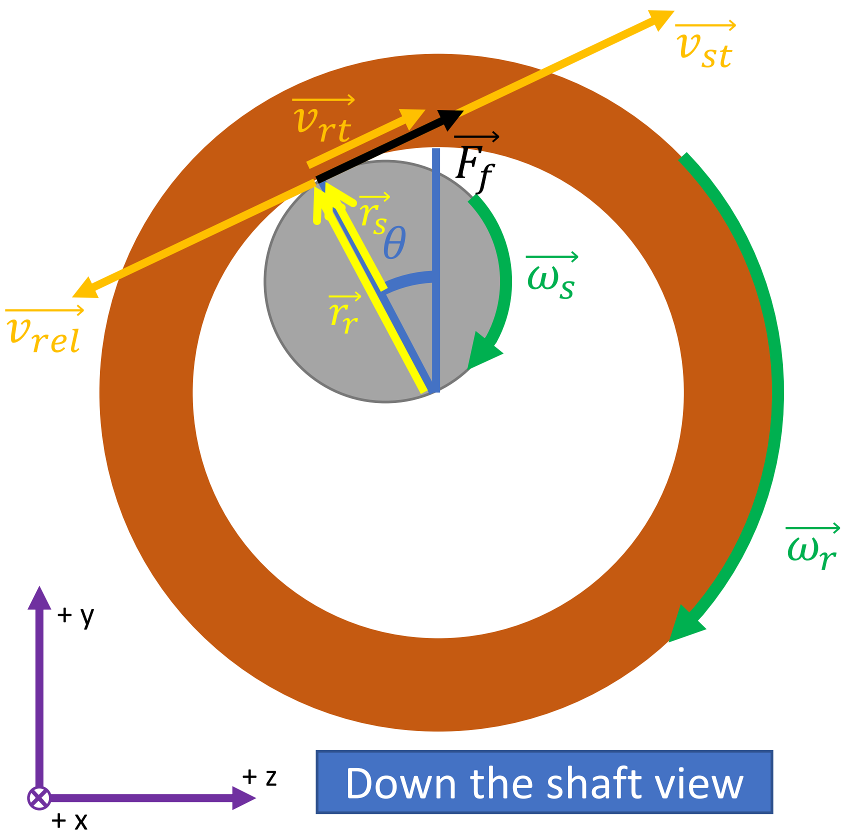


No movement along the shaft
No tilt

One-directional movement
Driving mechanism – Friction force

Oscillatory movement
Two types of oscillations

Relative velocity



$$\overrightarrow{v_{rt}} = \overrightarrow{\omega_r} \times \overrightarrow{r_r}$$

$$\overrightarrow{v_{st}} = \overrightarrow{\omega_s} \times \overrightarrow{r_s}$$

$$\overrightarrow{v_{rel}} = \overrightarrow{v_{rt}} - \overrightarrow{v_{st}}$$

Since $\overrightarrow{F_f} \parallel -\overrightarrow{v_{rel}}$, the ring speeds up

until:

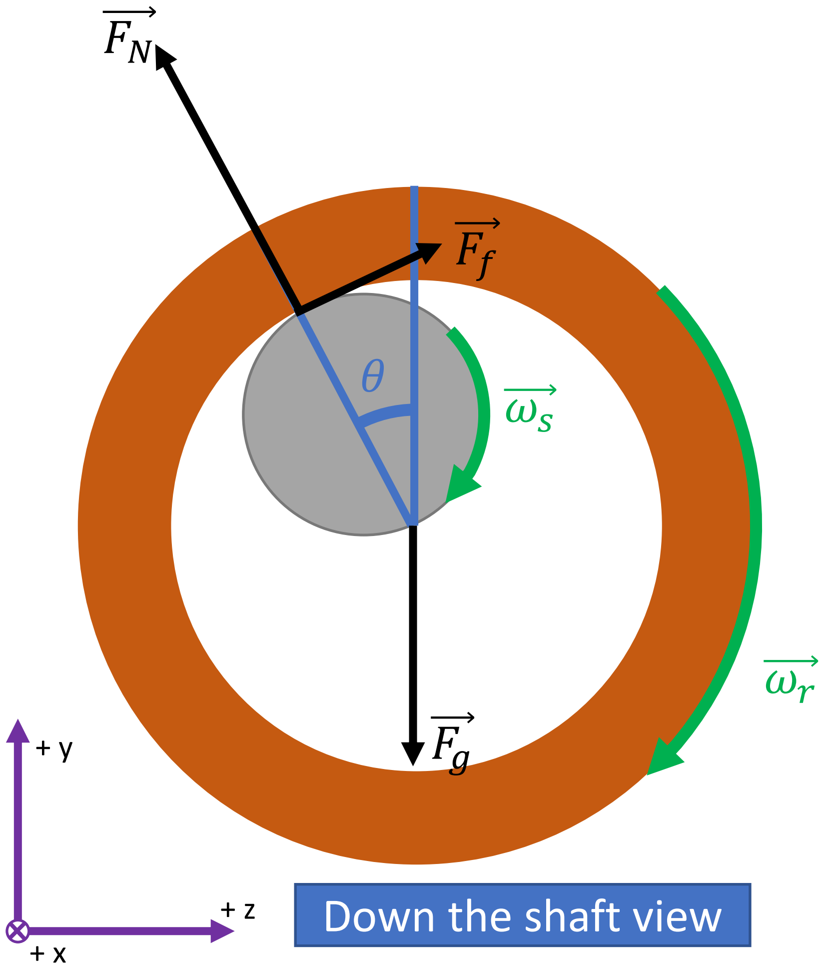
$$\overrightarrow{v_{rt}} = \overrightarrow{v_{st}}$$

$$\overrightarrow{\omega_r} \times \overrightarrow{r_r} = \overrightarrow{\omega_s} \times \overrightarrow{r_s}$$

$\overrightarrow{v_{rt}}$ = ring tangential velocity

$\overrightarrow{v_{st}}$ = shaft tangential velocity

Force balance



$$F_g = m_r g$$

$$F_N = m_r g \cos\theta$$

$$F_f = \mu m_r g \cos\theta$$

Initially:

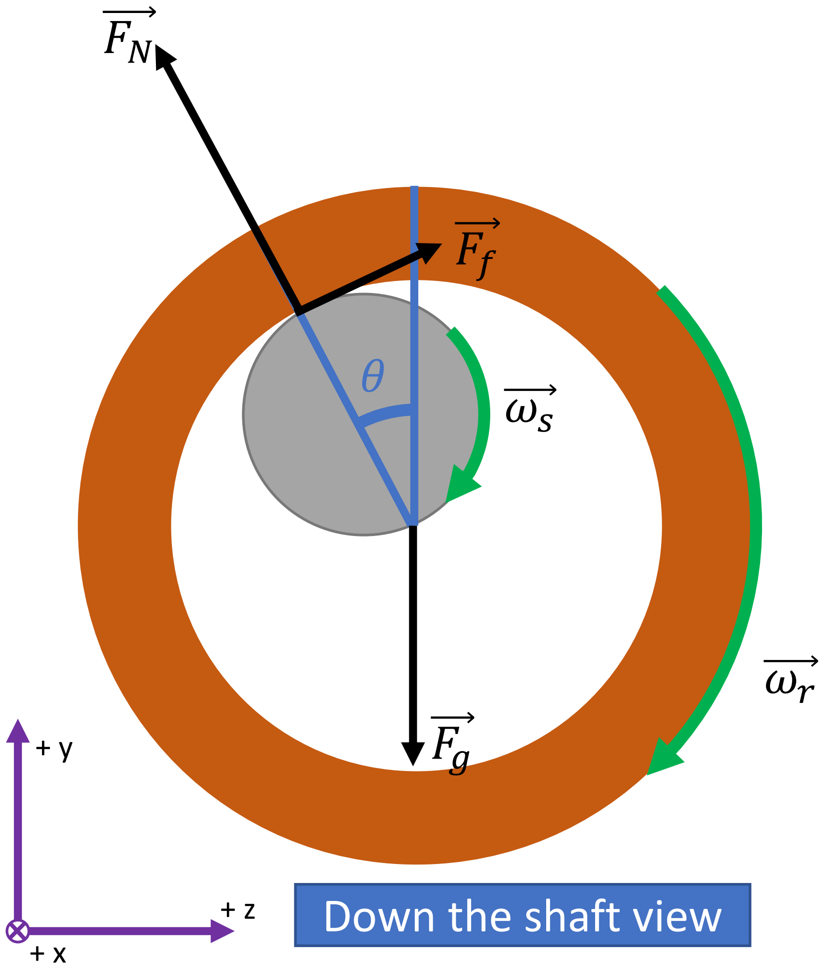
$$\theta = 0, \omega_r = 0$$

After ring is placed on shaft:

$$\sum F_y = F_N \cos\theta - F_G + F_F \sin\theta = m a_y$$

$$\sum F_z = F_F \cos\theta - F_N \sin\theta = m a_z$$

Force balance



$$\sum F_y = F_N \cos\theta - F_G + F_F \sin\theta = ma_y$$

$$a_y = g \sin\theta\,(\mu \cos\theta - \sin\theta)$$

$$\sum F_z = F_f \cos\theta - F_N \sin\theta = ma_z$$

$$a_z = g \cos\theta\,(\mu \cos\theta - \sin\theta)$$

When $a_z, a_y = 0$:

$$\boxed{\mu_k = \tan\theta}$$

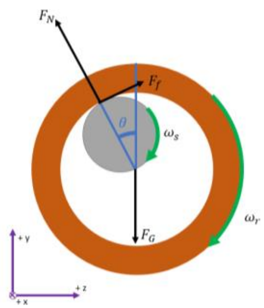
Measured Value – Coefficient of Friction

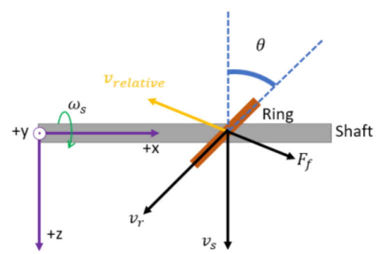
Trial #	$\theta \pm 0.1°$
1	5.9
2	5.7
3	6.1
4	6.3
5	6.2
6	6.0
7	6.1
8	6.3
9	6.1

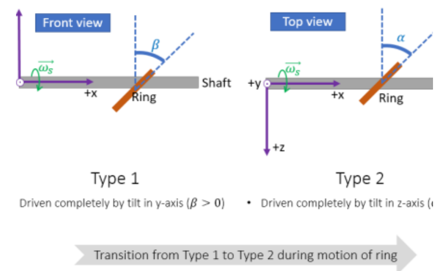
$$\theta = 6.1 \pm 0.2°$$
$$\mu = \tan\theta$$

$$\mu_k = 0.11 \pm 0.01$$
(cardboard on aluminum with WD-40)

Theoretical Model





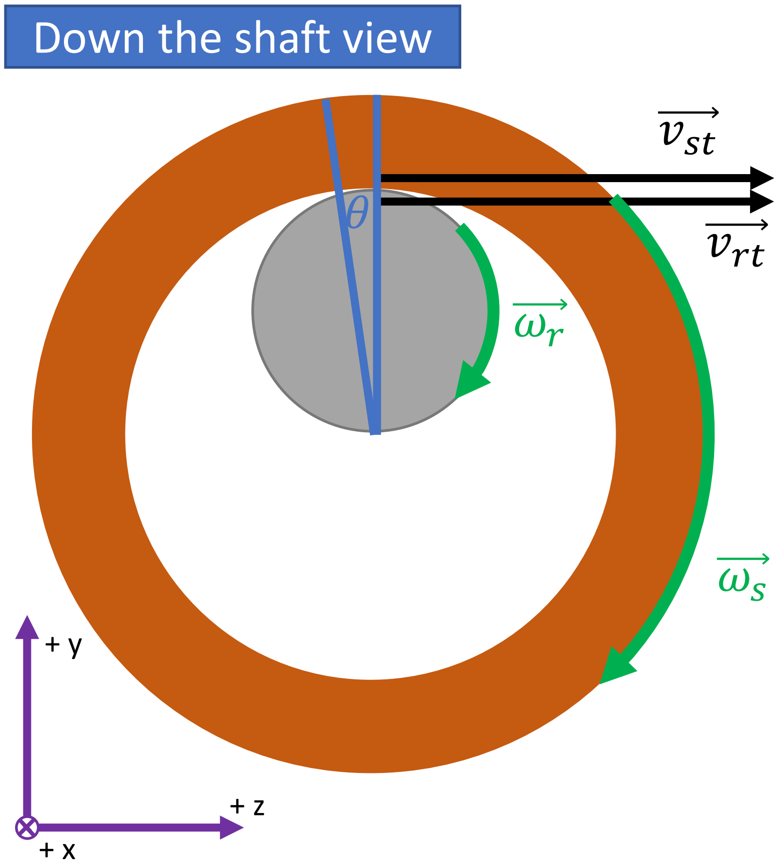


No movement along shaft
No tilt

One-directional movement
Driving mechanism - tilt in the y-axis

Oscillatory movement
Driving **mechanism** – tilt in the z-axis

Temporary Assumption

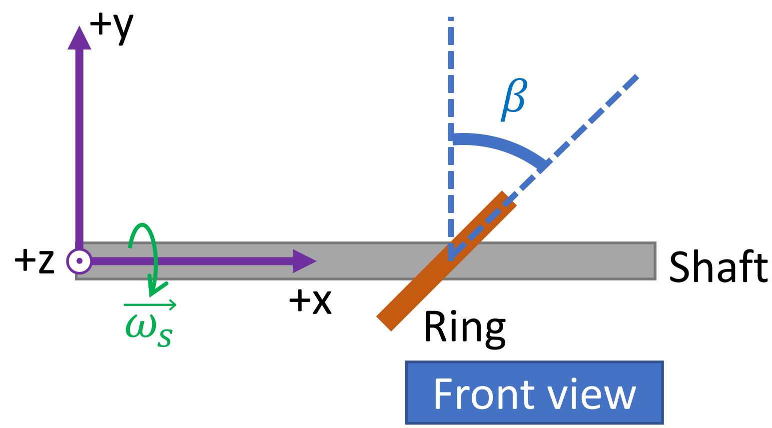


$$v_{rt} = v_{st}$$

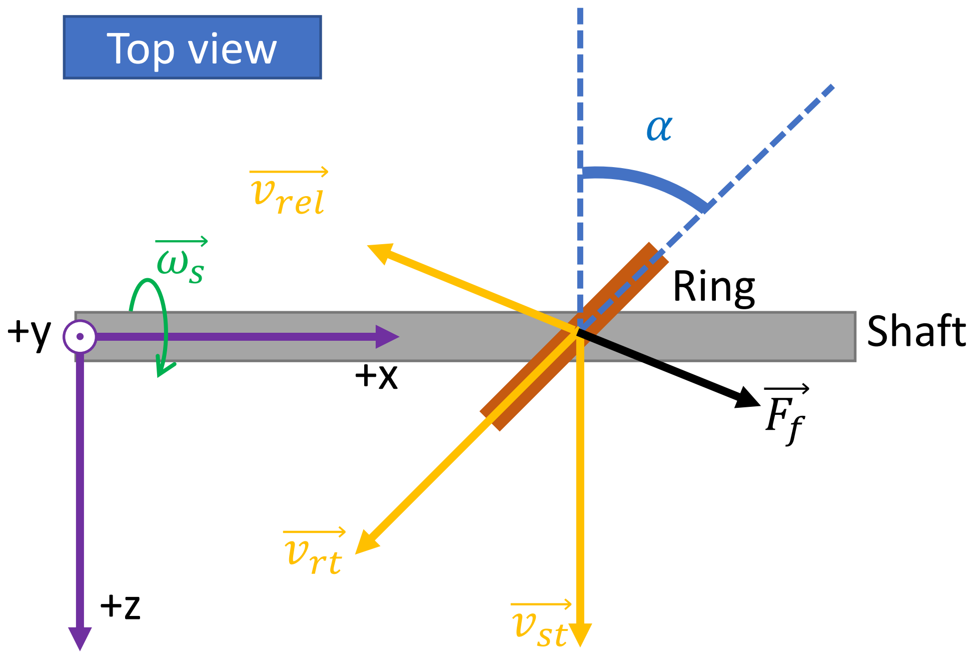
$$v_r = 0$$

$$\beta = 0$$

$$\theta \approx 0$$



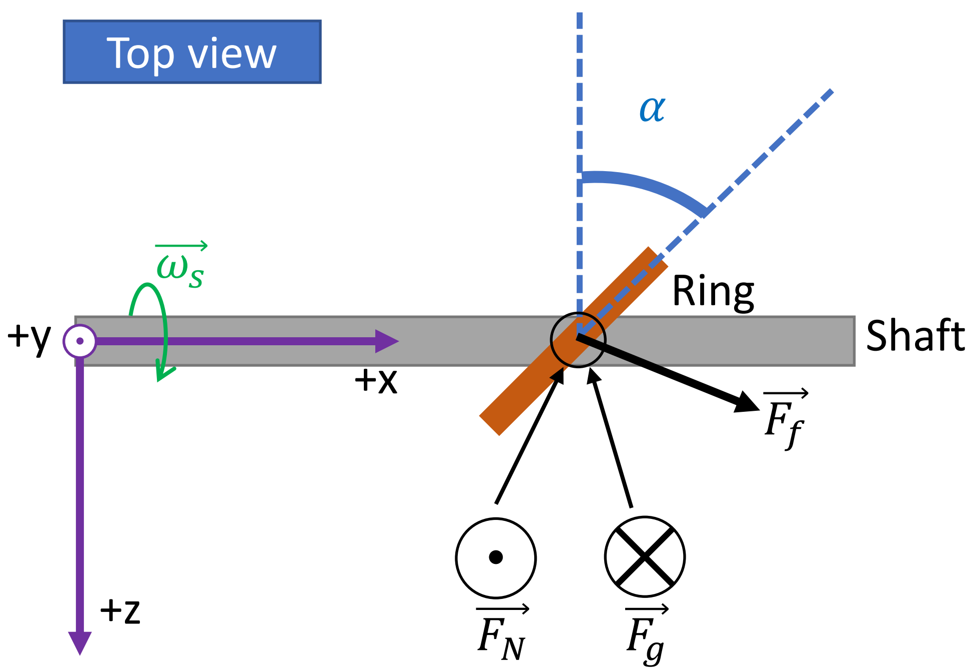
Relative Velocity



$$\overrightarrow{v_{rel}} = \overrightarrow{v_{rt}} - \overrightarrow{v_{st}}$$

$$\overrightarrow{F_f} \parallel -\overrightarrow{v_{rel}}$$

Force balance



$$\sum F_y = F_N - F_G = 0 = m_r a_y$$

$$a_y = 0$$

$$\sum F_z = F_f \sin\alpha = m_r a_z$$

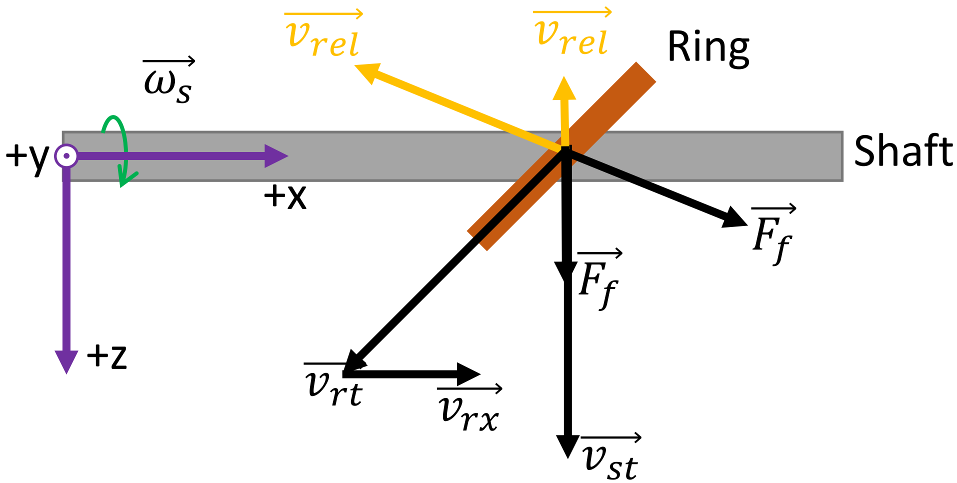
$$a_z = \mu g \sin\frac{\alpha}{2}$$

$$\sum F_x = F_f \cos\theta = m_r a_x$$

$$a_x = \mu g \cos\frac{\alpha}{2}$$

Changing relative velocity

Top view



Center of mass now has an acceleration in the x direction:

$$v_{rx}(t) = v_0 + \int_0^t a_x(t)dt$$

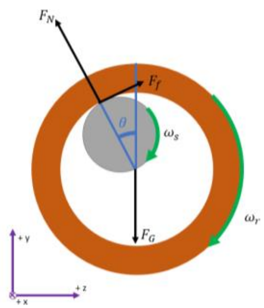
$$v_0 = 0$$

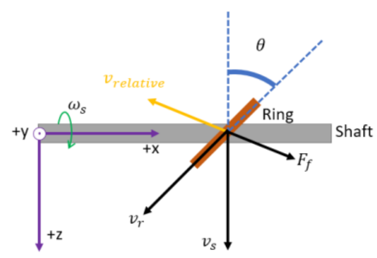
$$\vec{v}_{rel} = \vec{v}_{rt} + \vec{v}_{rx} - \vec{v}_{st}$$

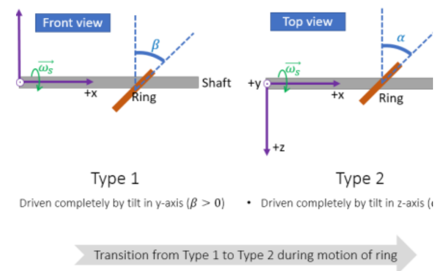
As v_{rx} increases, the x-component of F_f decreases, and a_x decreases:

Therefore, v_{rx} has a maximum velocity.

Theoretical Model





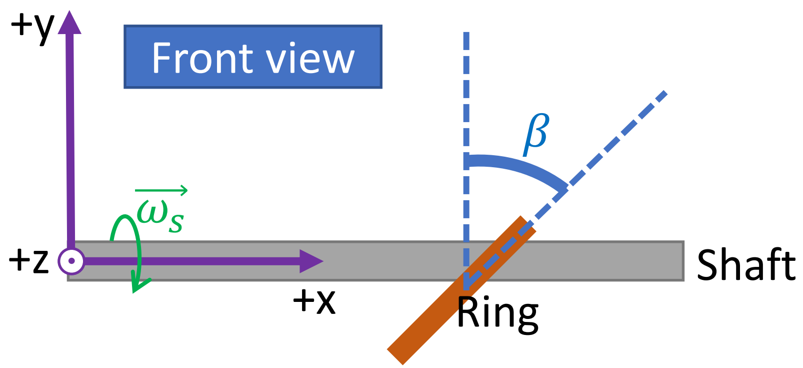


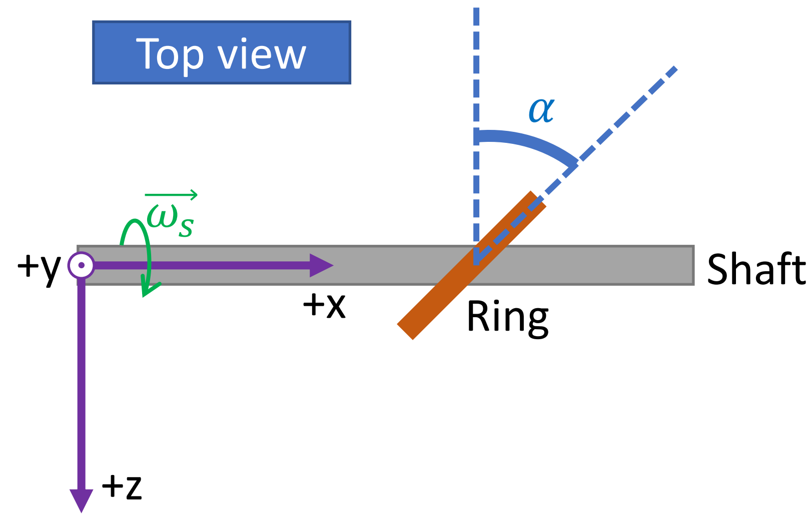
No movement along the shaft
No tilt

One-directional movement
Driving mechanism – Friction force

Oscillatory movement
Two types of oscillations

Types of Oscillations





Type 1

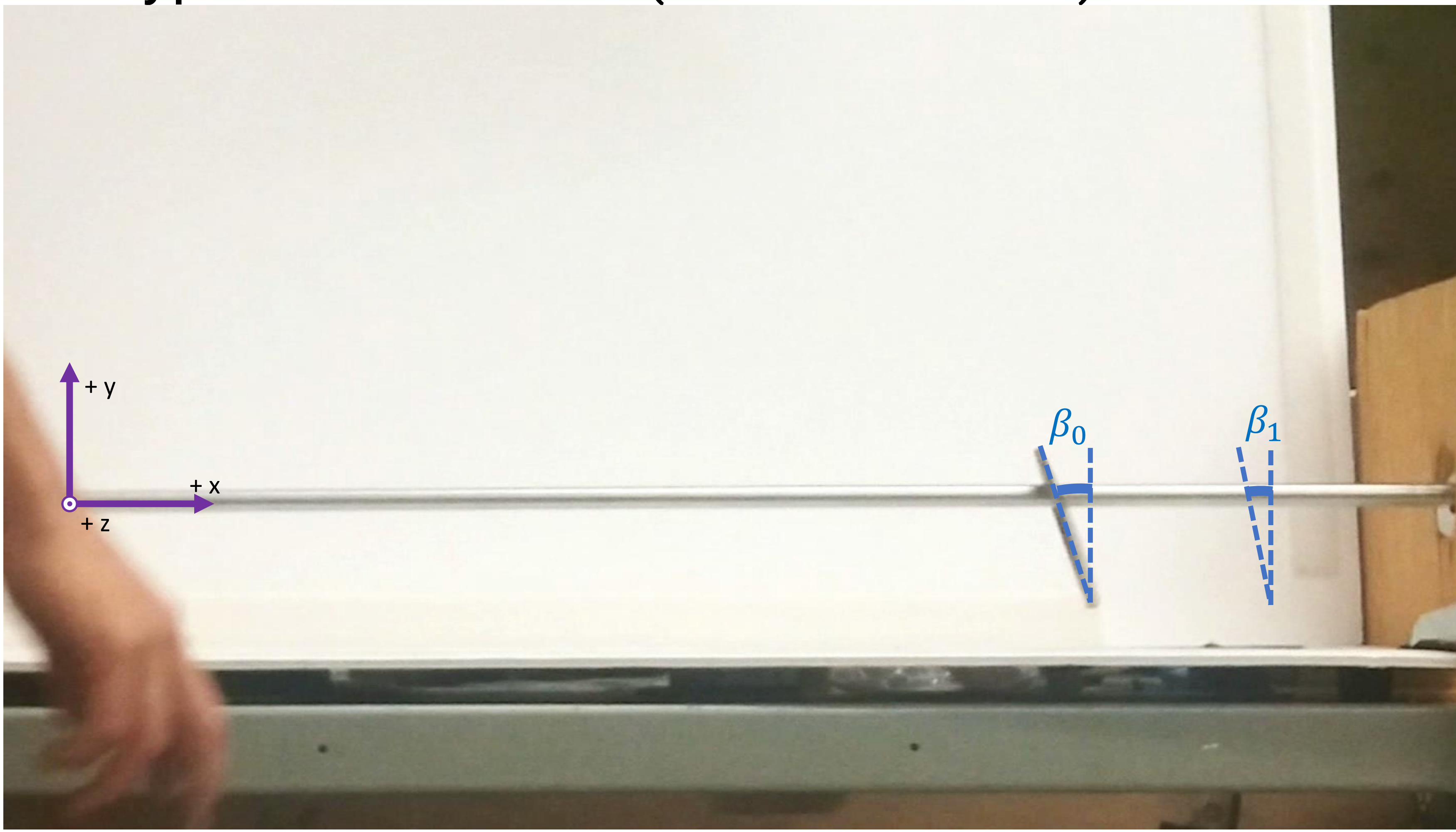
- Driven completely by tilt in y-axis ($\beta > 0$)

Type 2

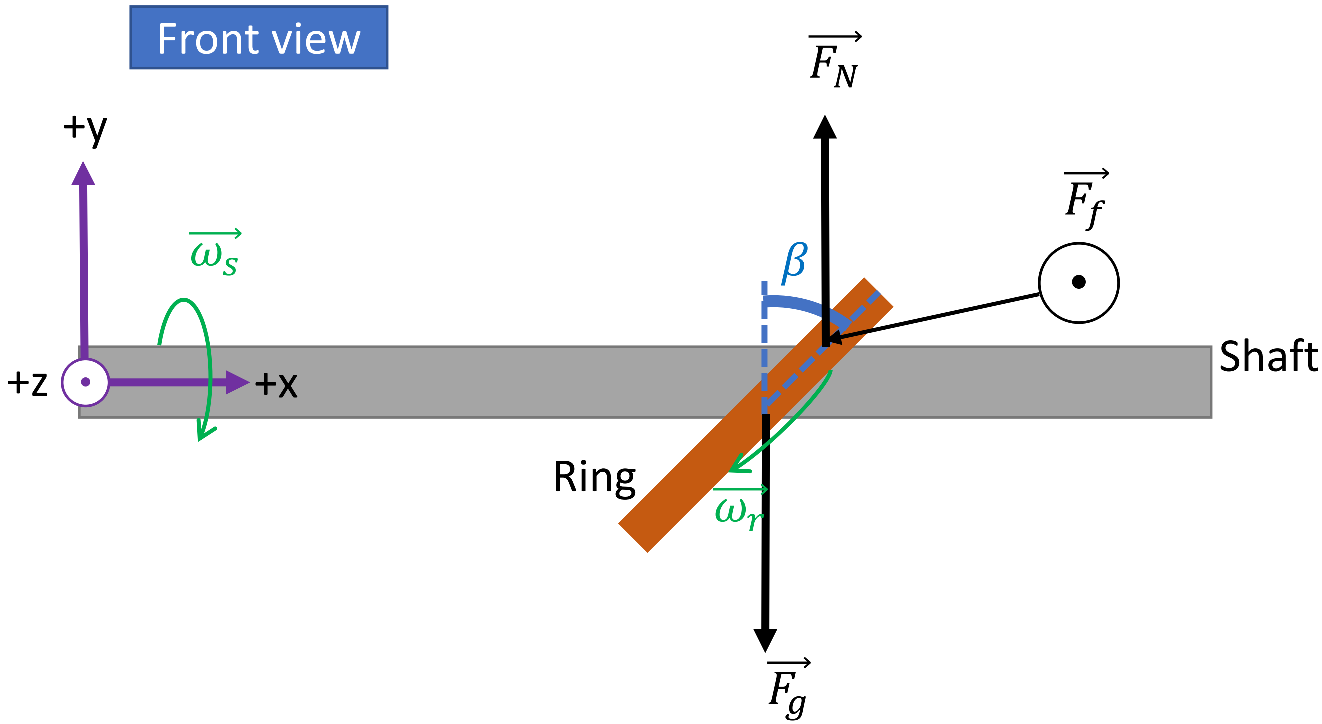
- Driven completely by tilt in z-axis ($\alpha > 0$)

Transition from Type 1 to Type 2 during motion of ring

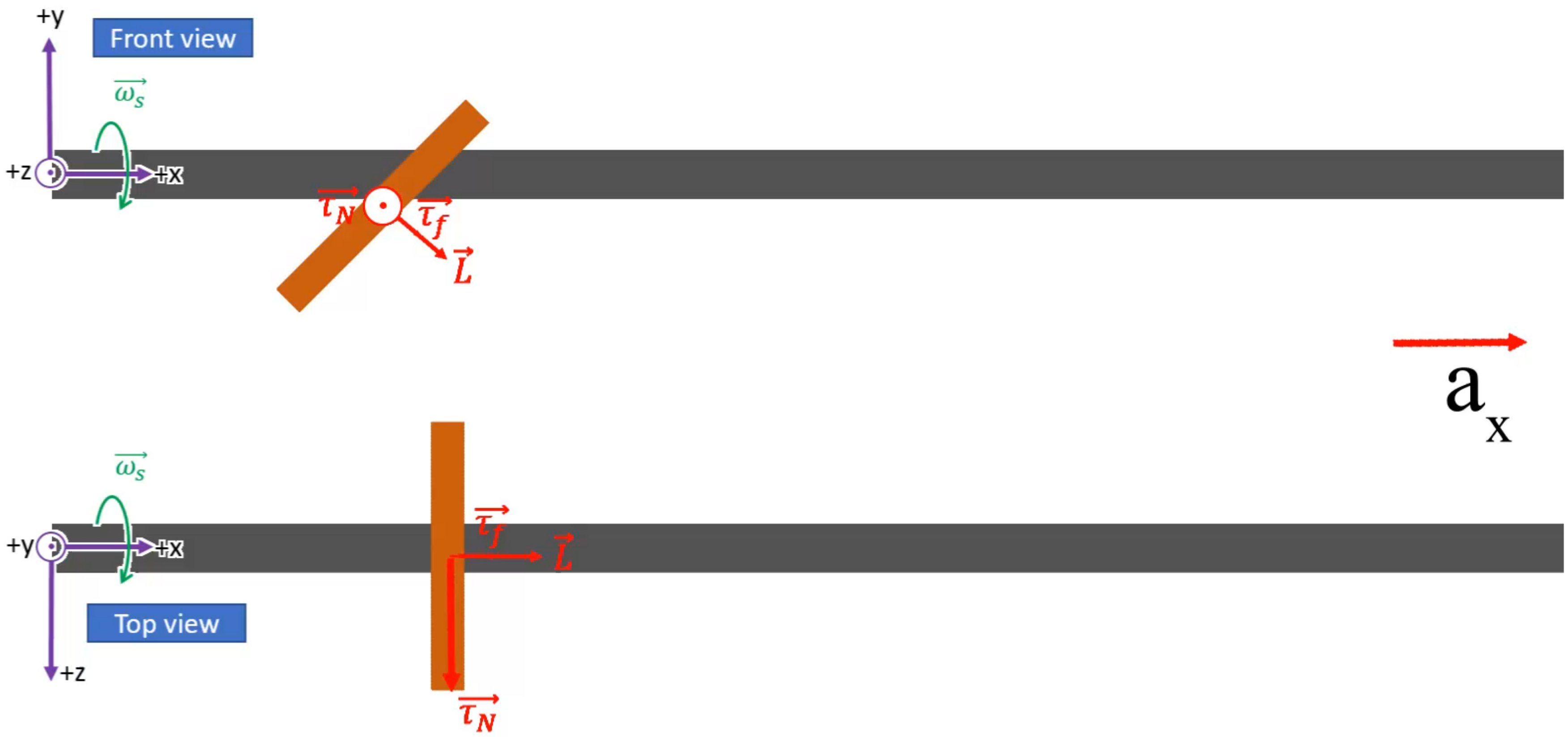
Type 1 Oscillations (tilt in the z-axis)



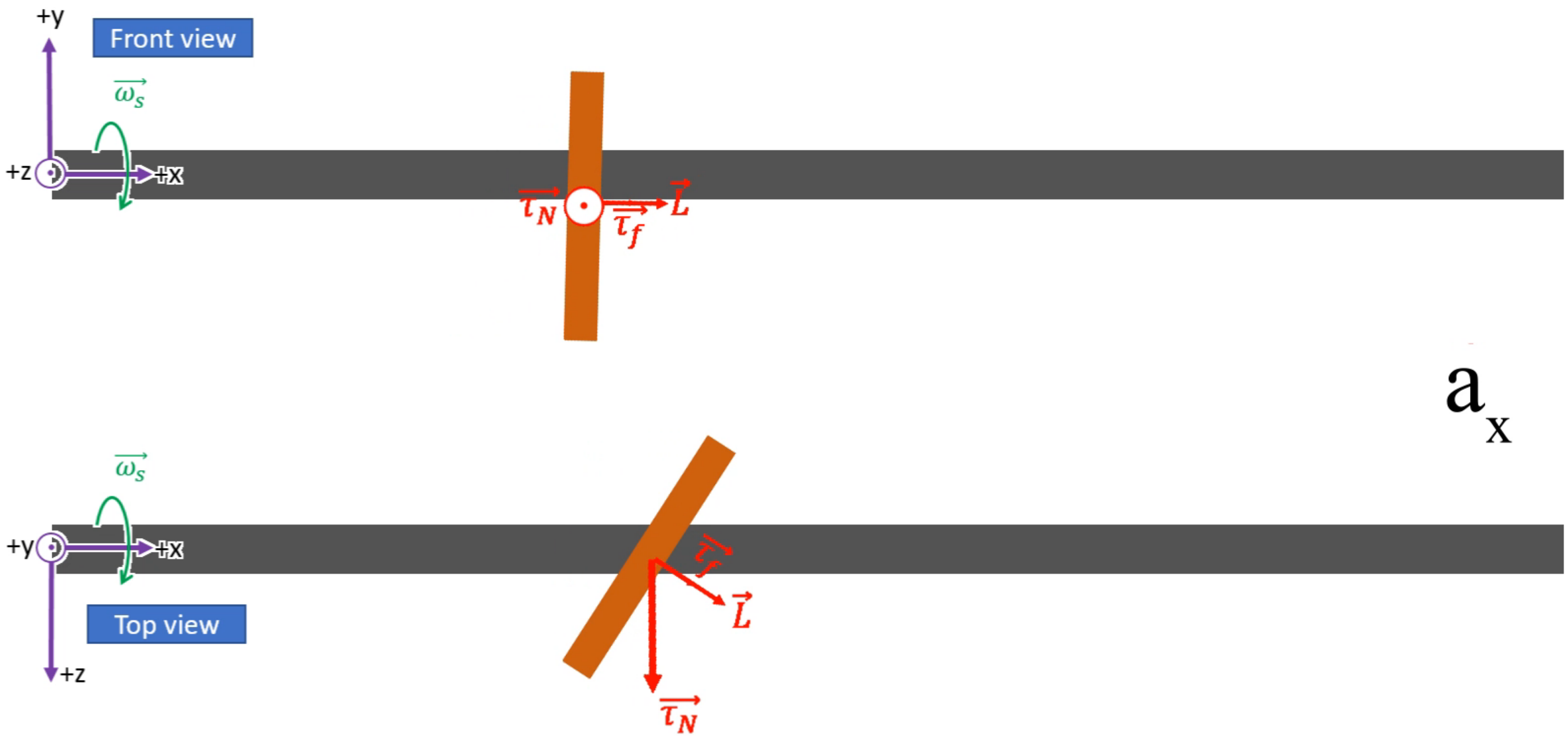
Type 1 Oscillations (tilt in the z-axis)



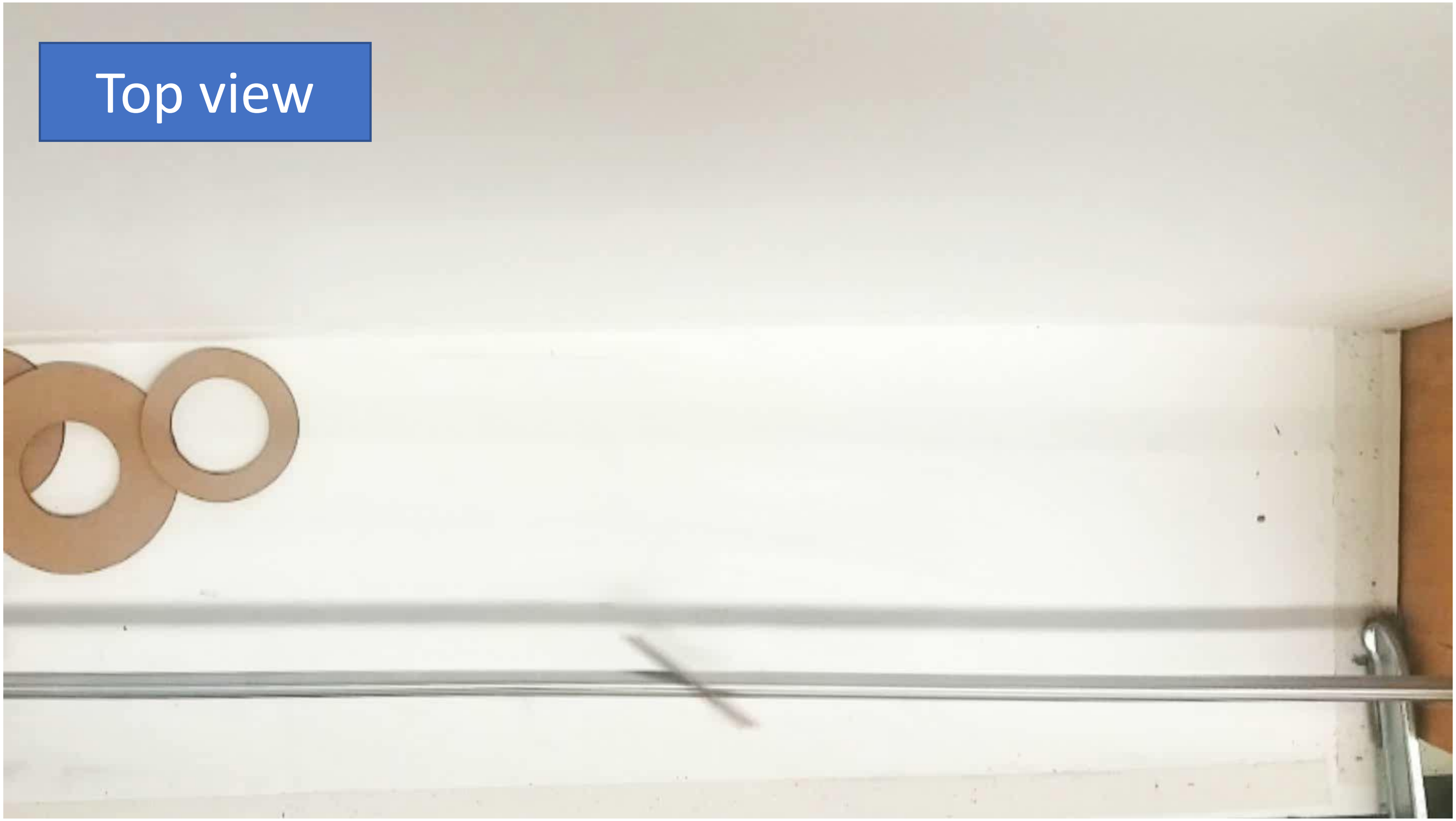
Type 1 Oscillations (tilt in the z-axis)



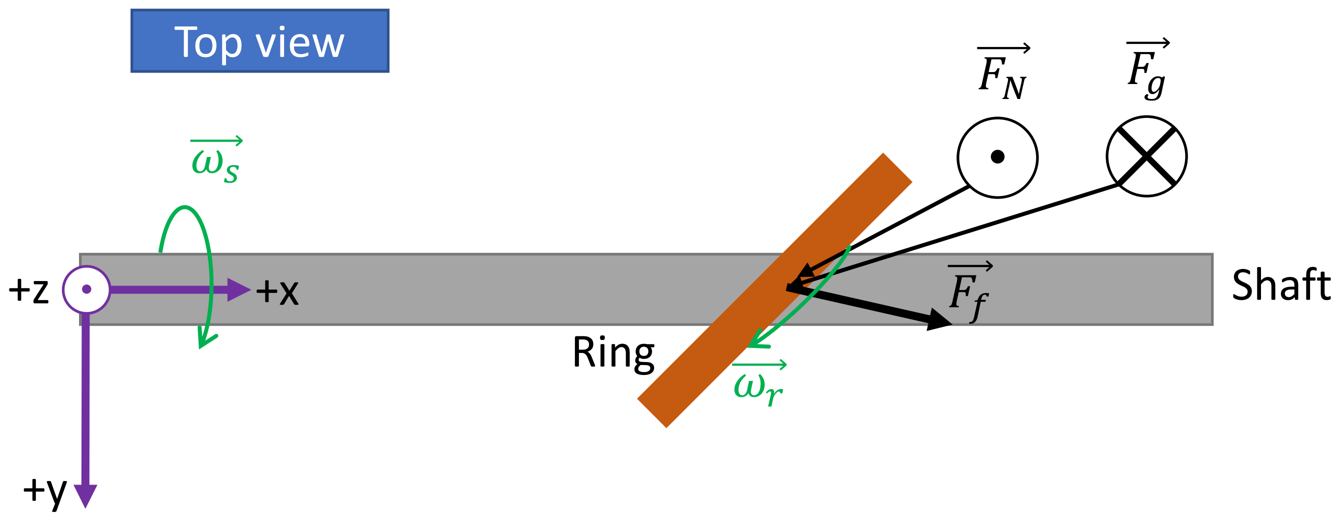
Type 1 Oscillations (tilt in the z-axis)



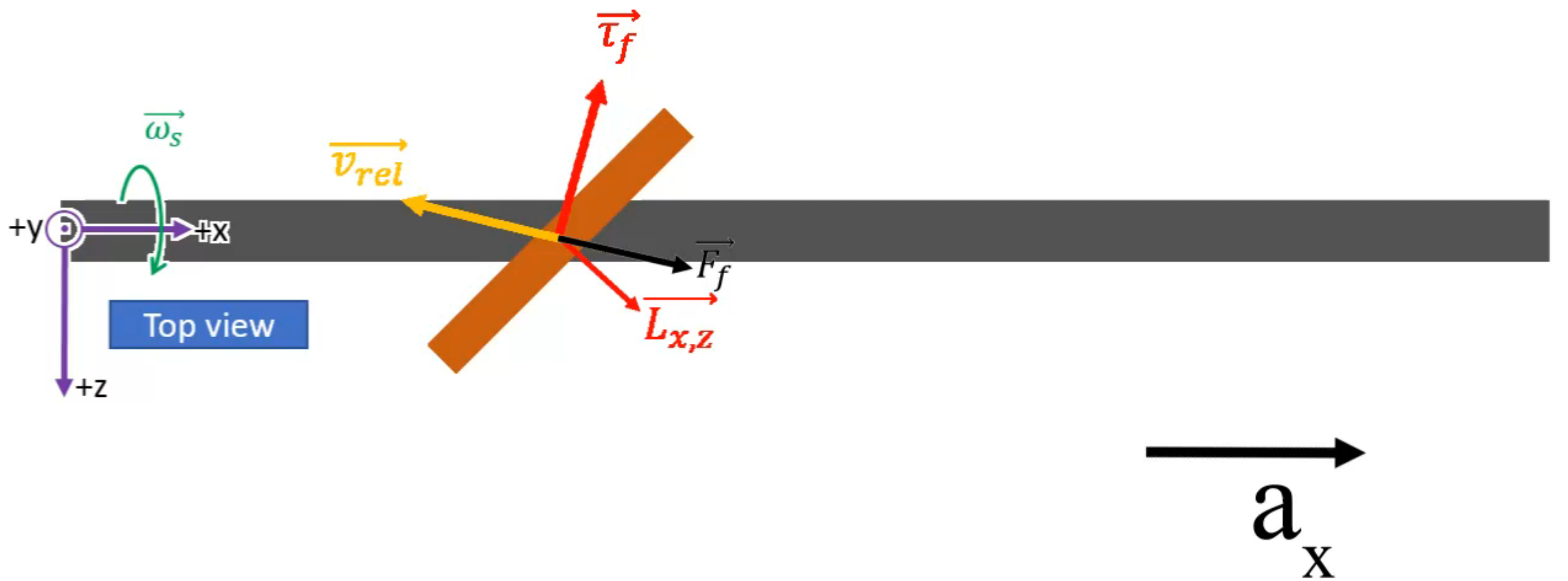
Type 2 Oscillations (tilt in the y-axis)



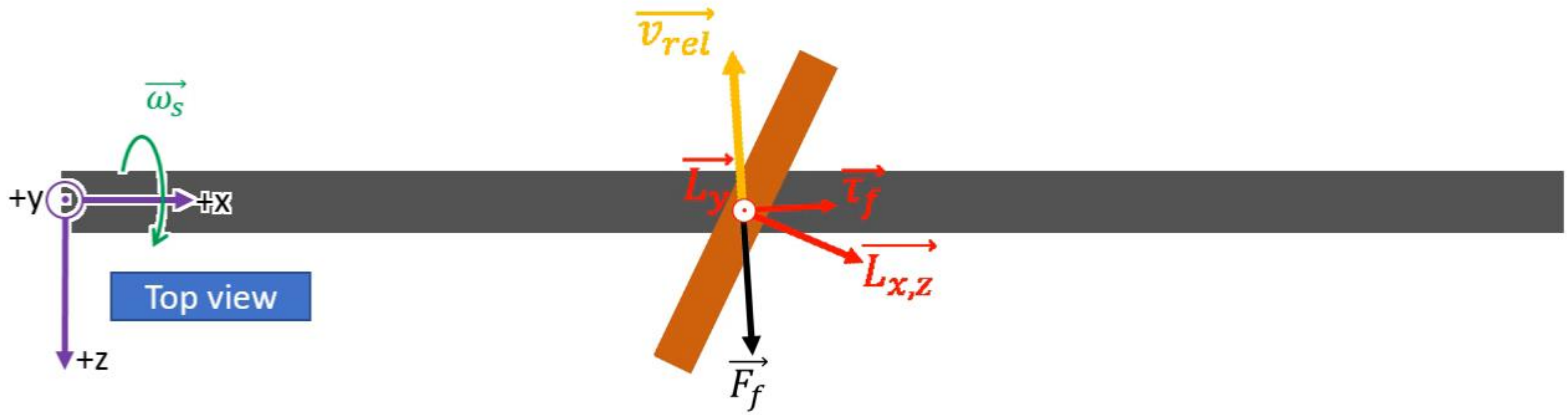
Type 2 Oscillations (tilt in the y-axis)



Type 2 Oscillations (tilt in the y-axis)

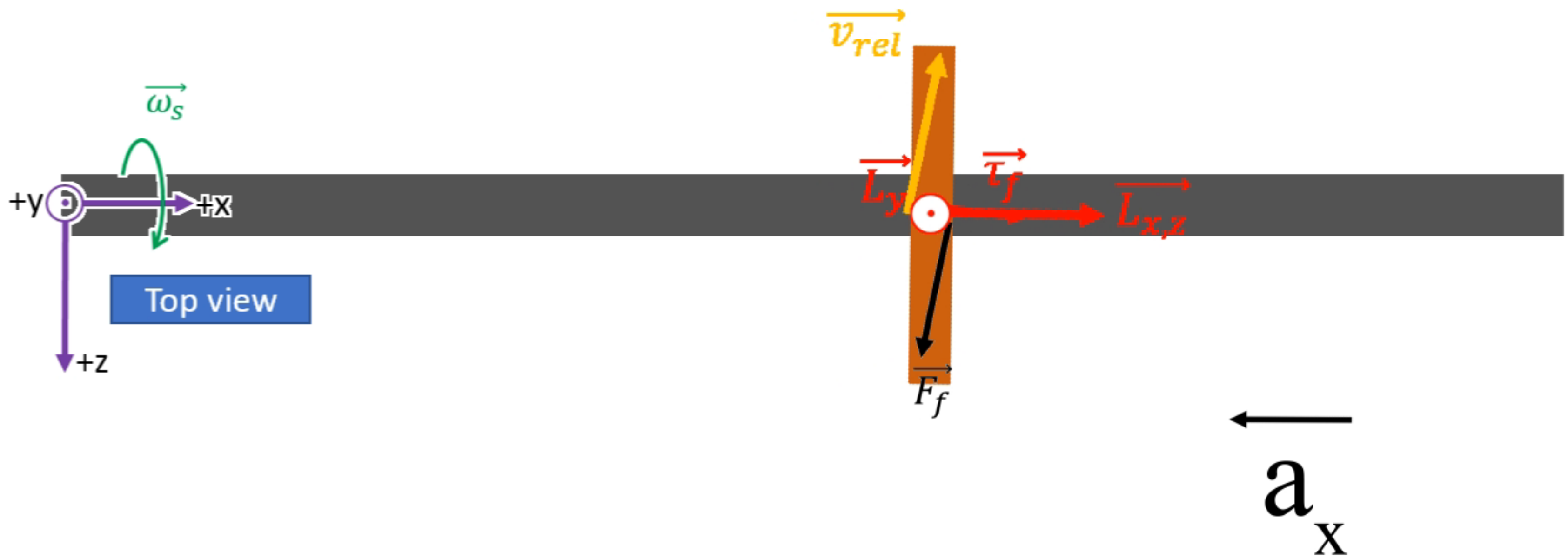


Type 2 Oscillations (tilt in the y-axis)



a_x

Type 2 Oscillations (tilt in the y-axis)



Transition – Type 1 → Type 2

Stage 1 (t = 0 to t = T/2)

- *Period is restricted by period of type 1 oscillation (Amplitude is lower)*

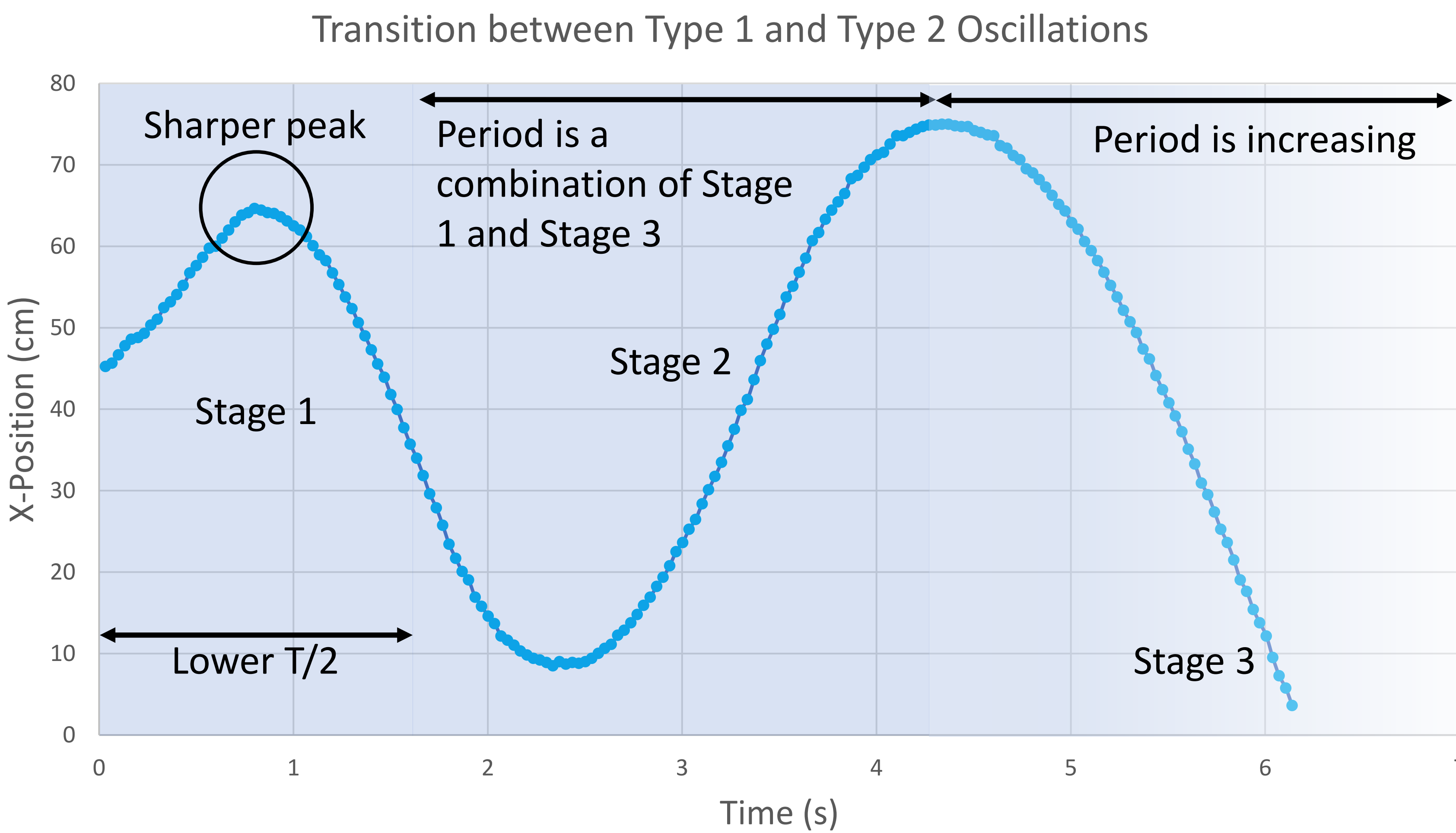
Stage 2 (t = T/2 to t = T)

- *Tilt in z-axis decreases*
  - *Lower restriction on period of motion (Higher amplitude)*

Stage 3 (t = T to t = infinity)

- *Motion of ring almost completely governed by type 2 oscillations*

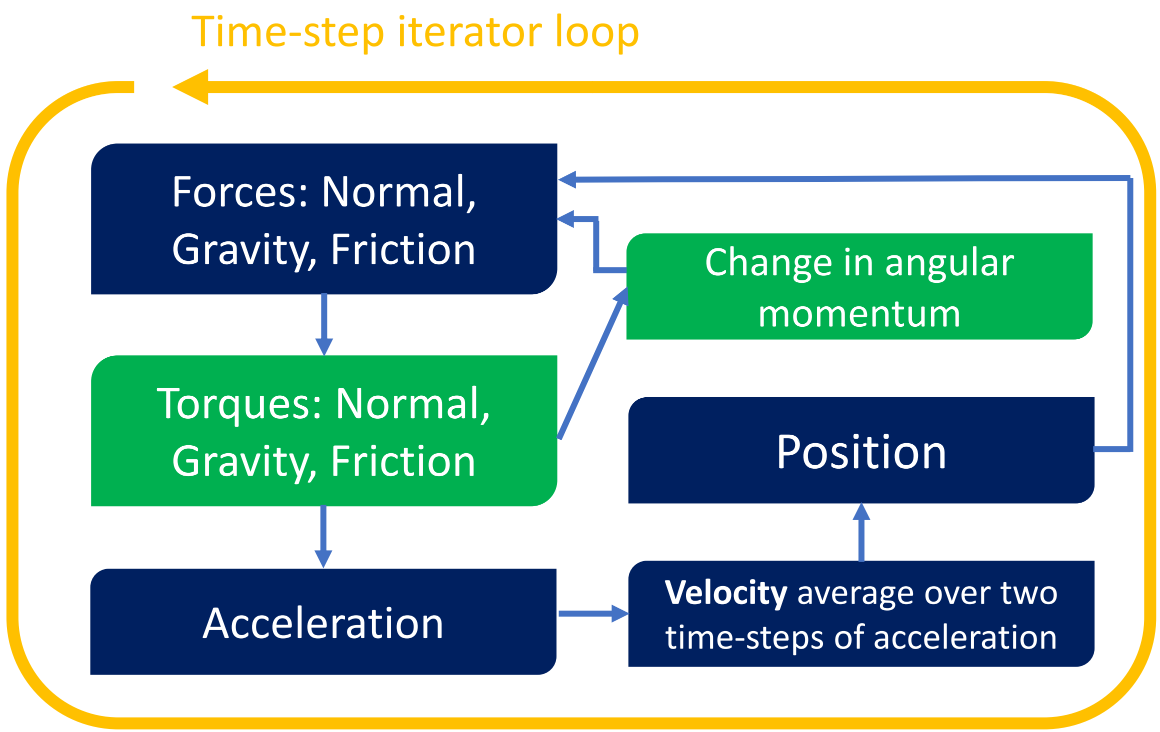
Transition – Type 1 → Type 2



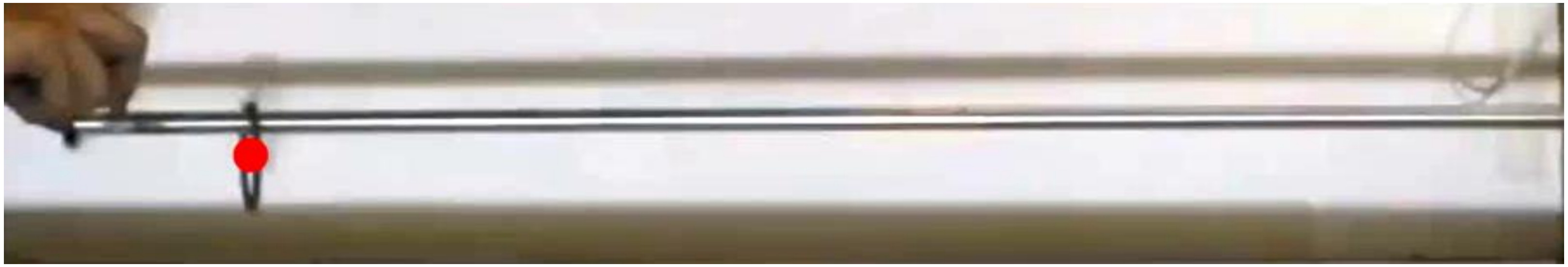
Simulation - Basic Principle

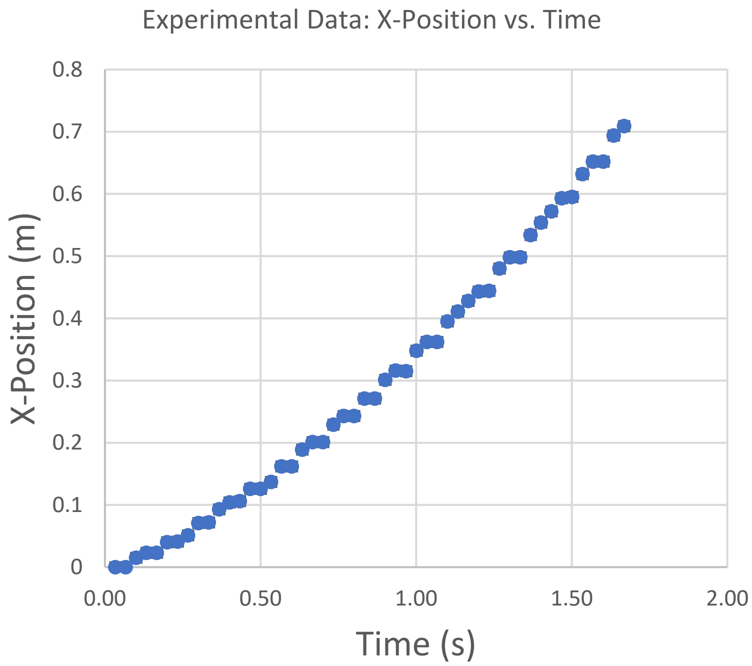
In order to combine the previous basic principles, the *Explicit Euler's Method* was used:

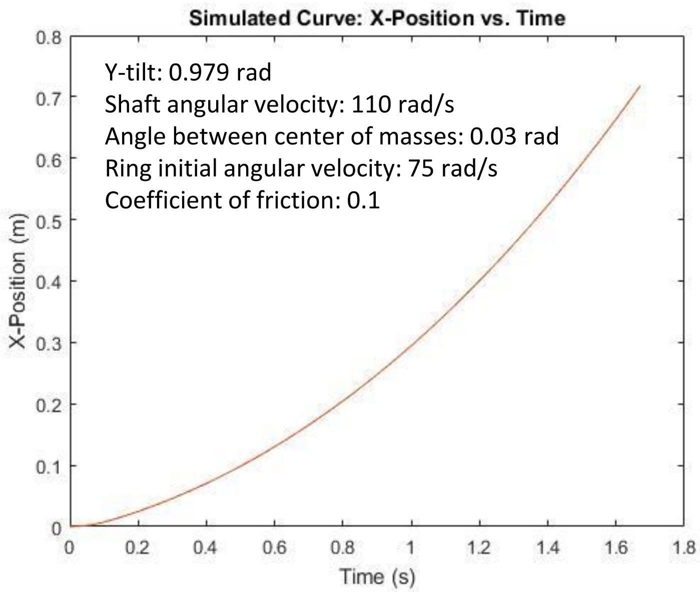
Constants:
- Ring mass
- Shaft angular velocity
- Coefficient of friction
- Inner ring radius
- Shaft radius



Experimental Verification (x vs. t)

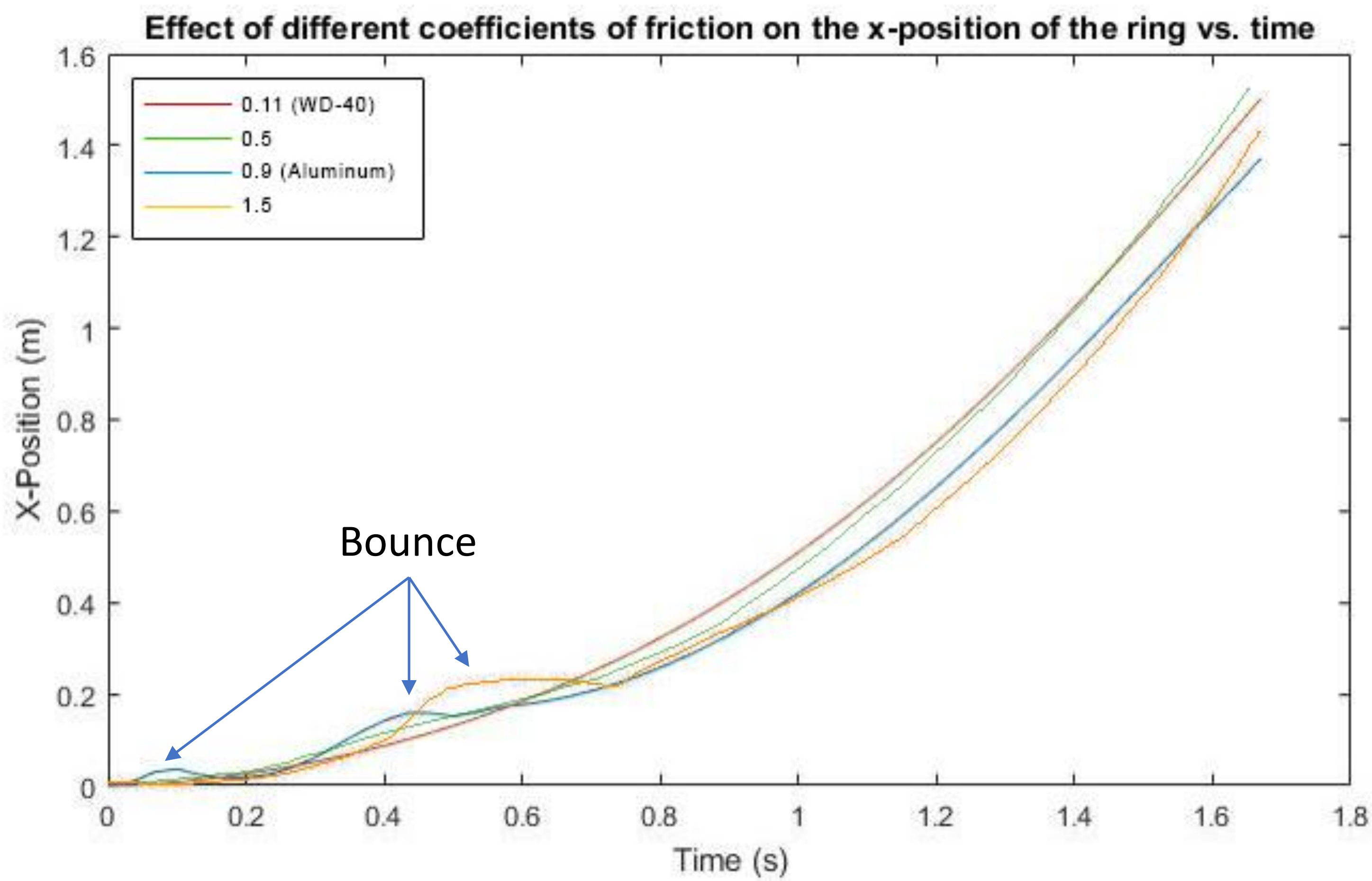






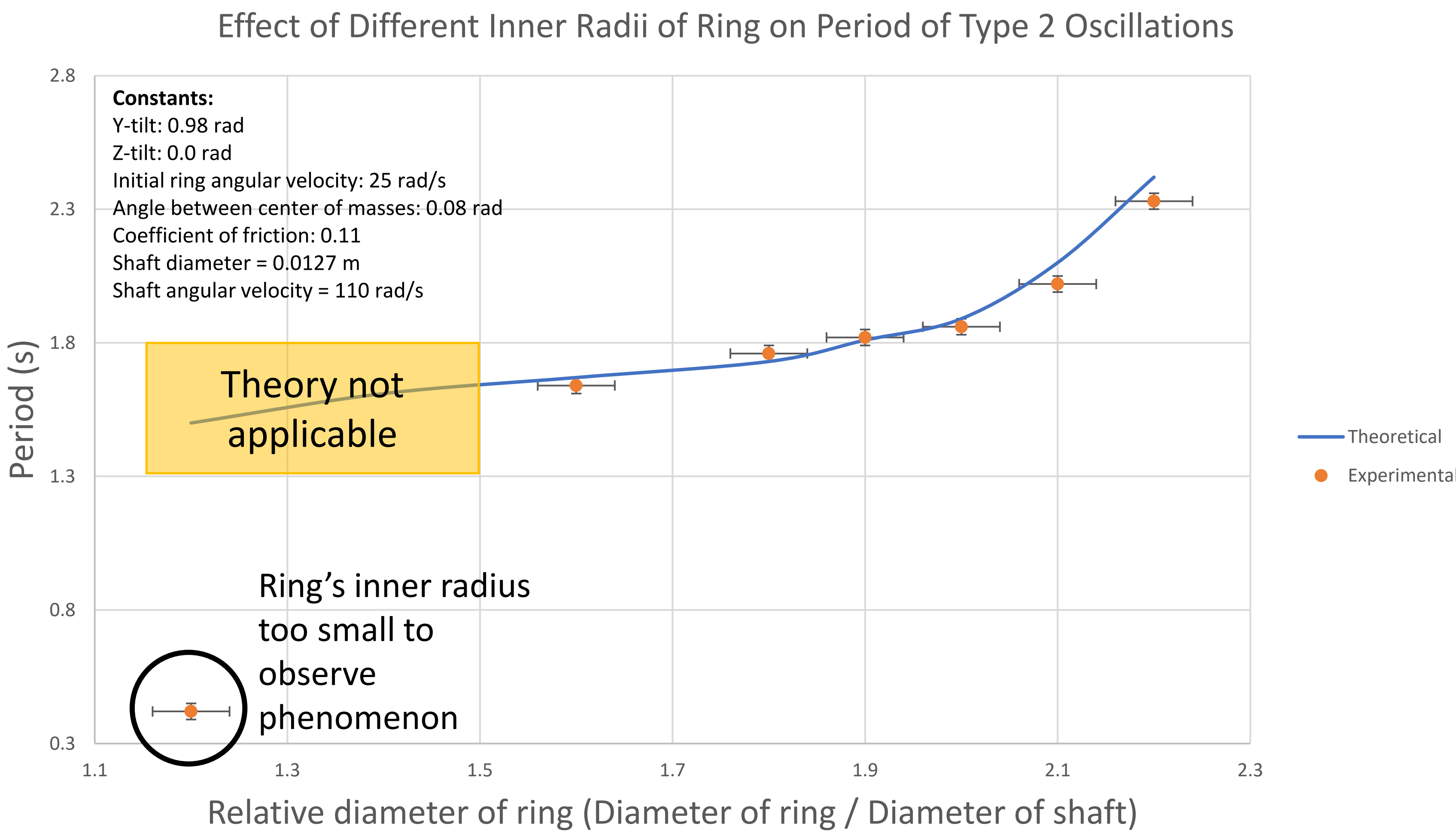
Key Parameters

Why use oil?

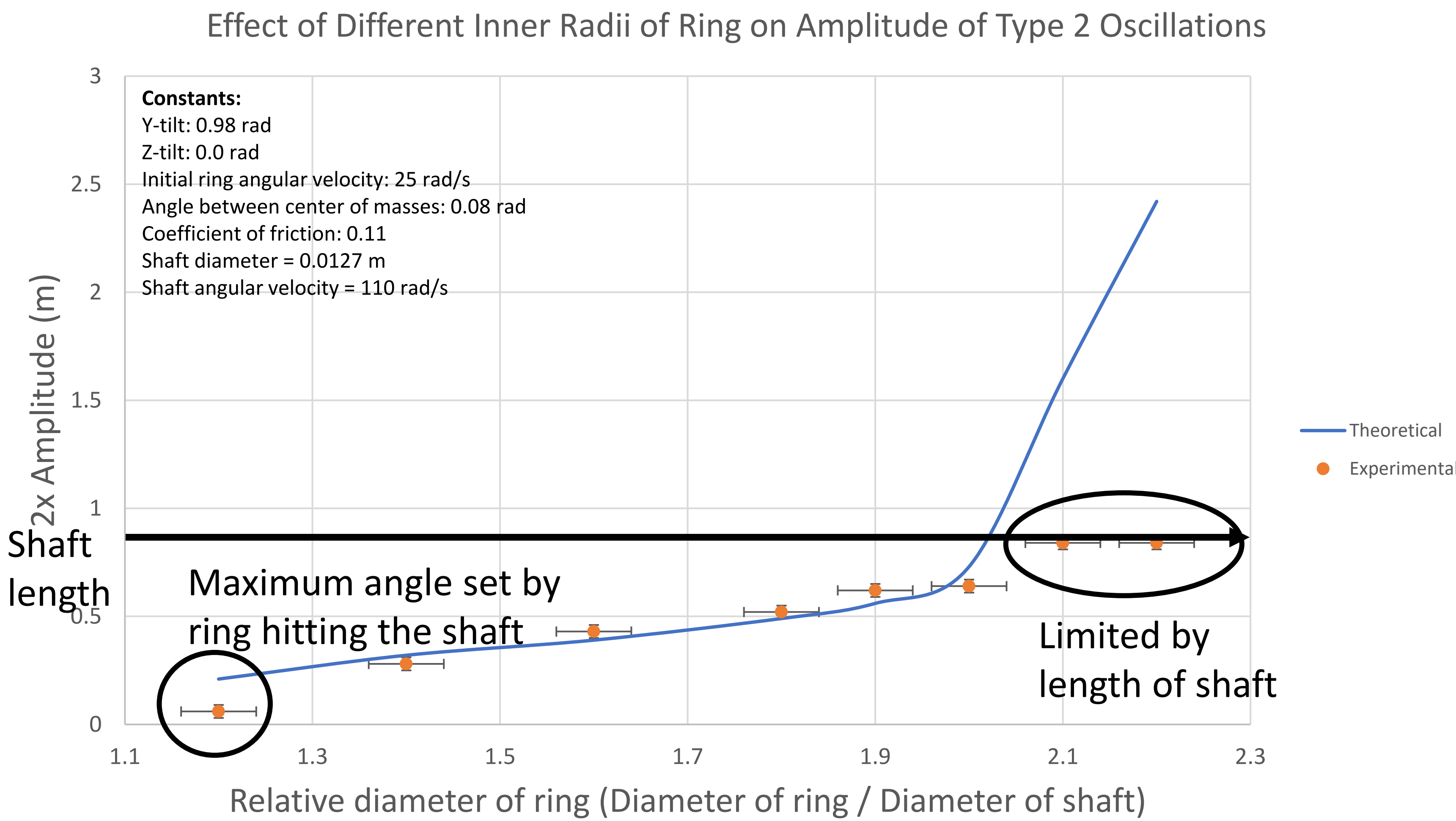

Effect of different coefficients of friction on the x-position of the ring vs. time
0.11 (WD-40)
0.5
0.9 (Aluminum)
1.5
Bounce
X-Position (m)
Time (s)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

Why use oil?

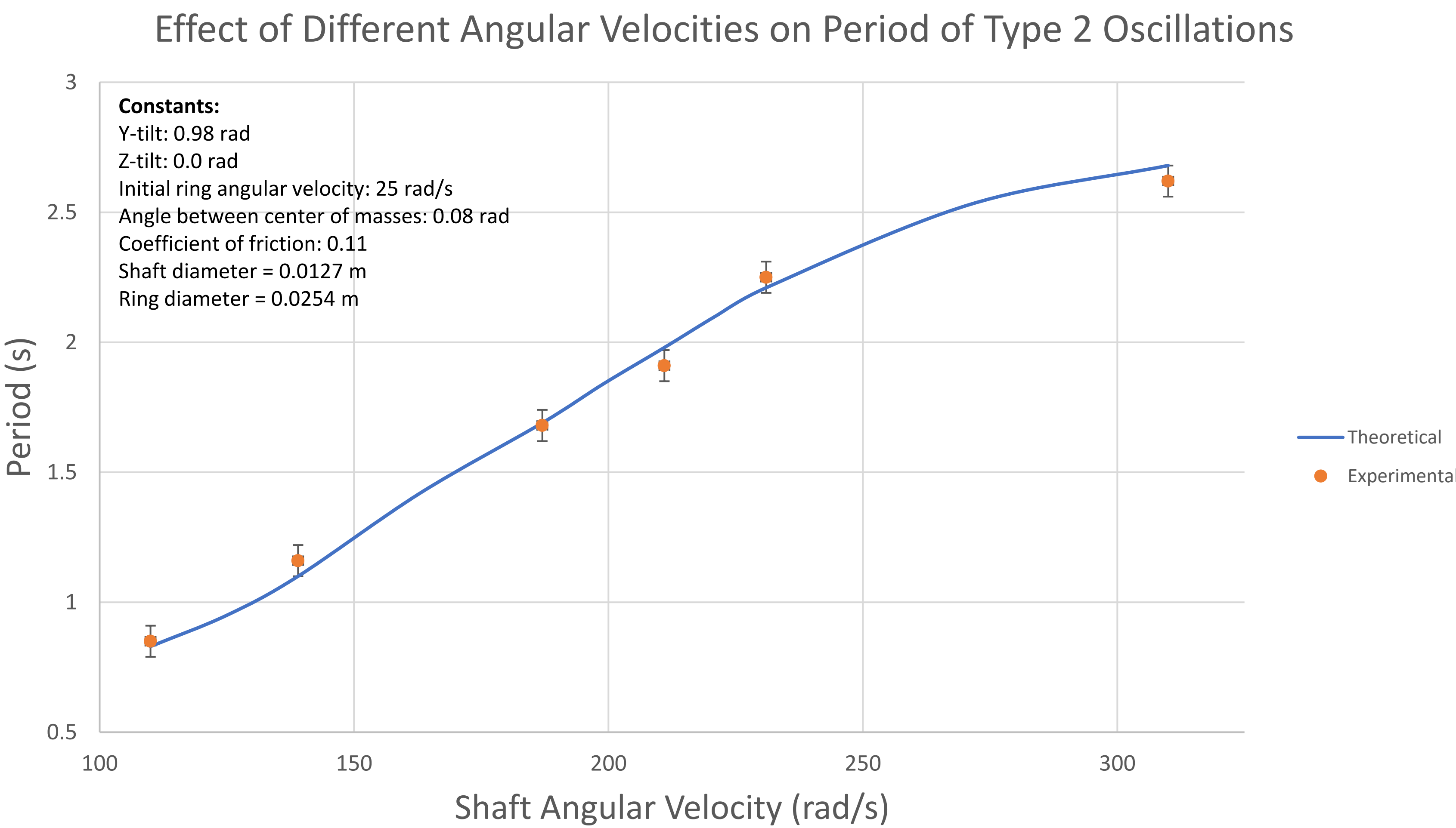
Different inner radii



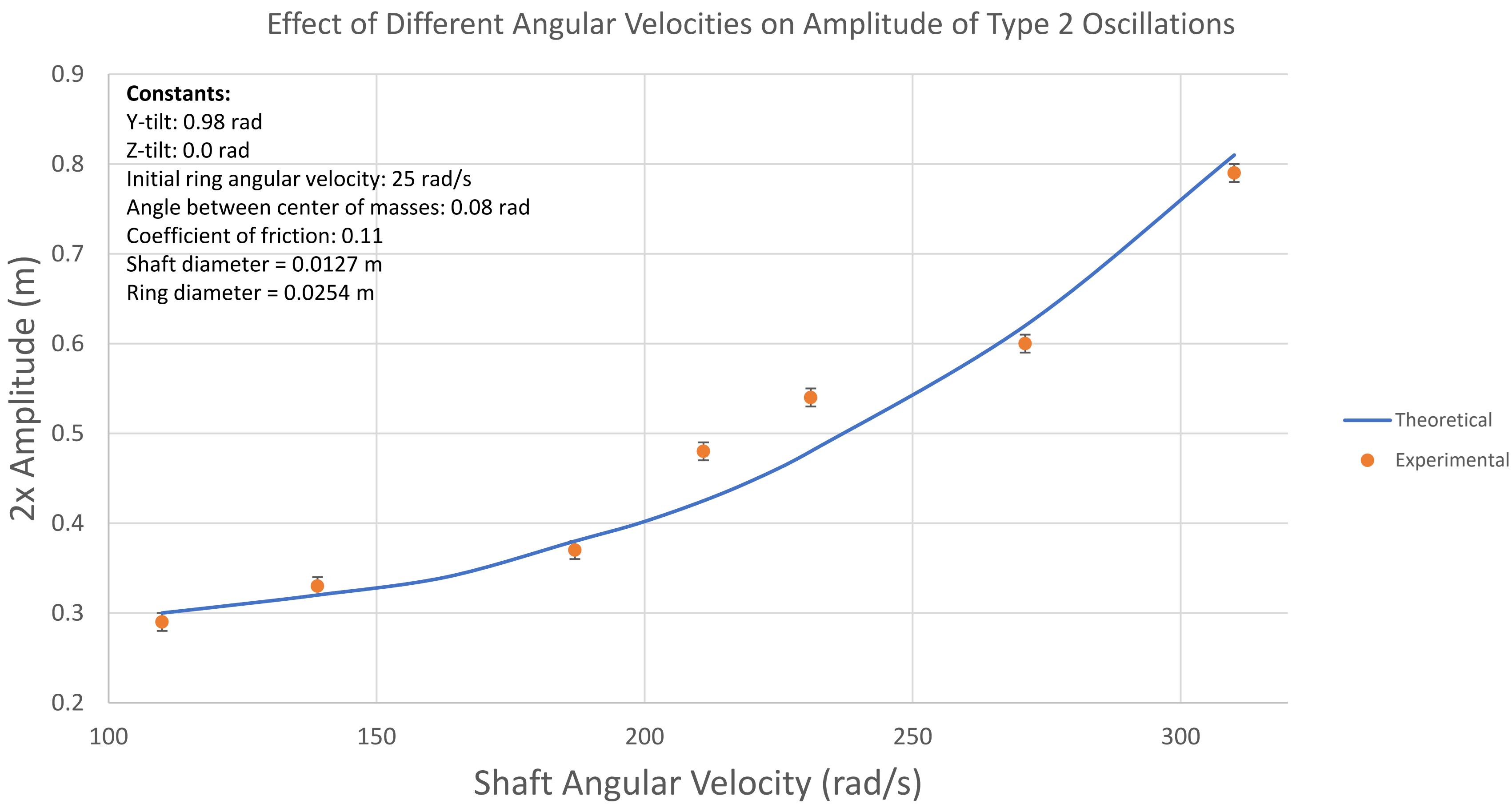
Different inner radii



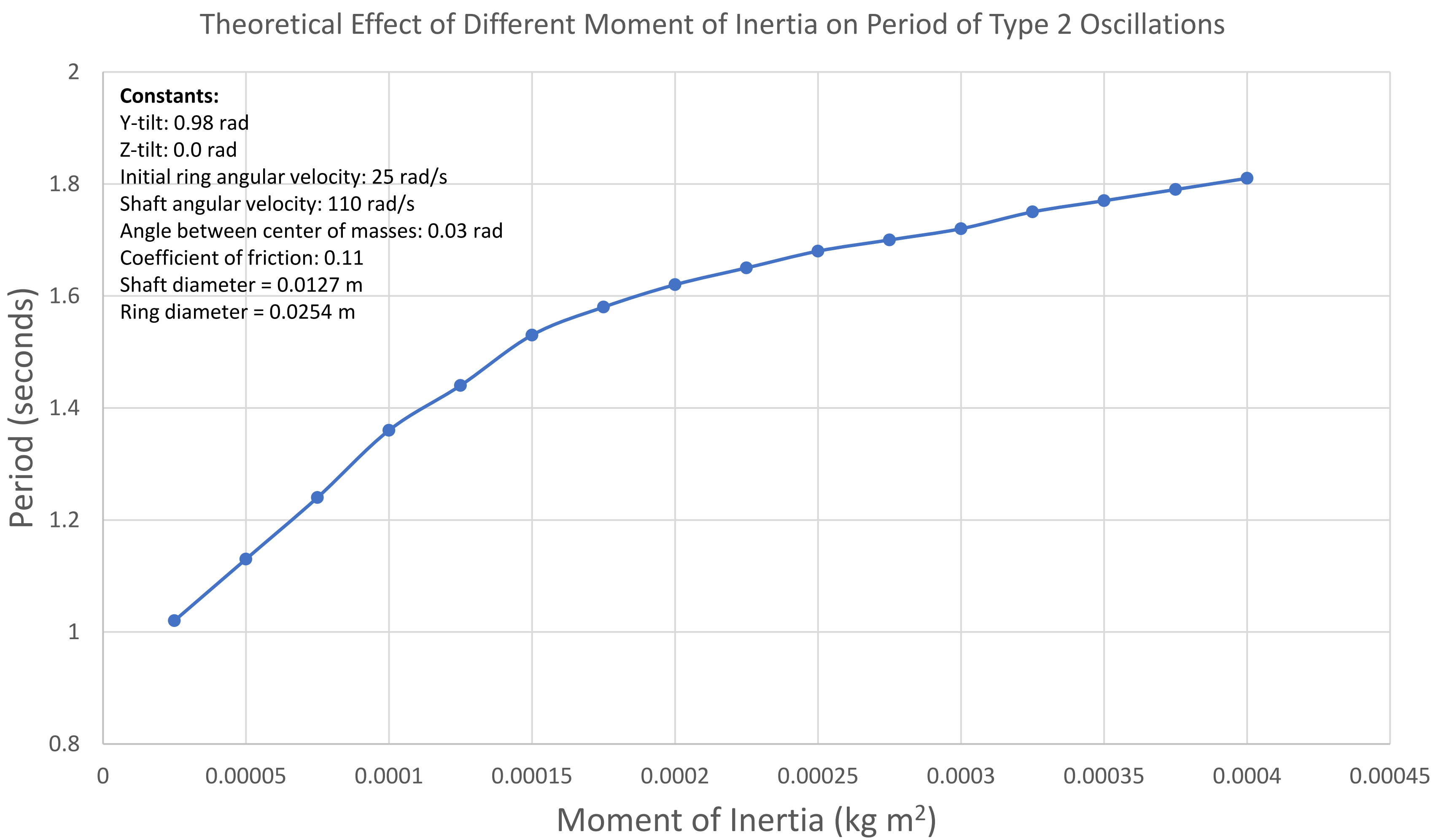
Different shaft angular velocities



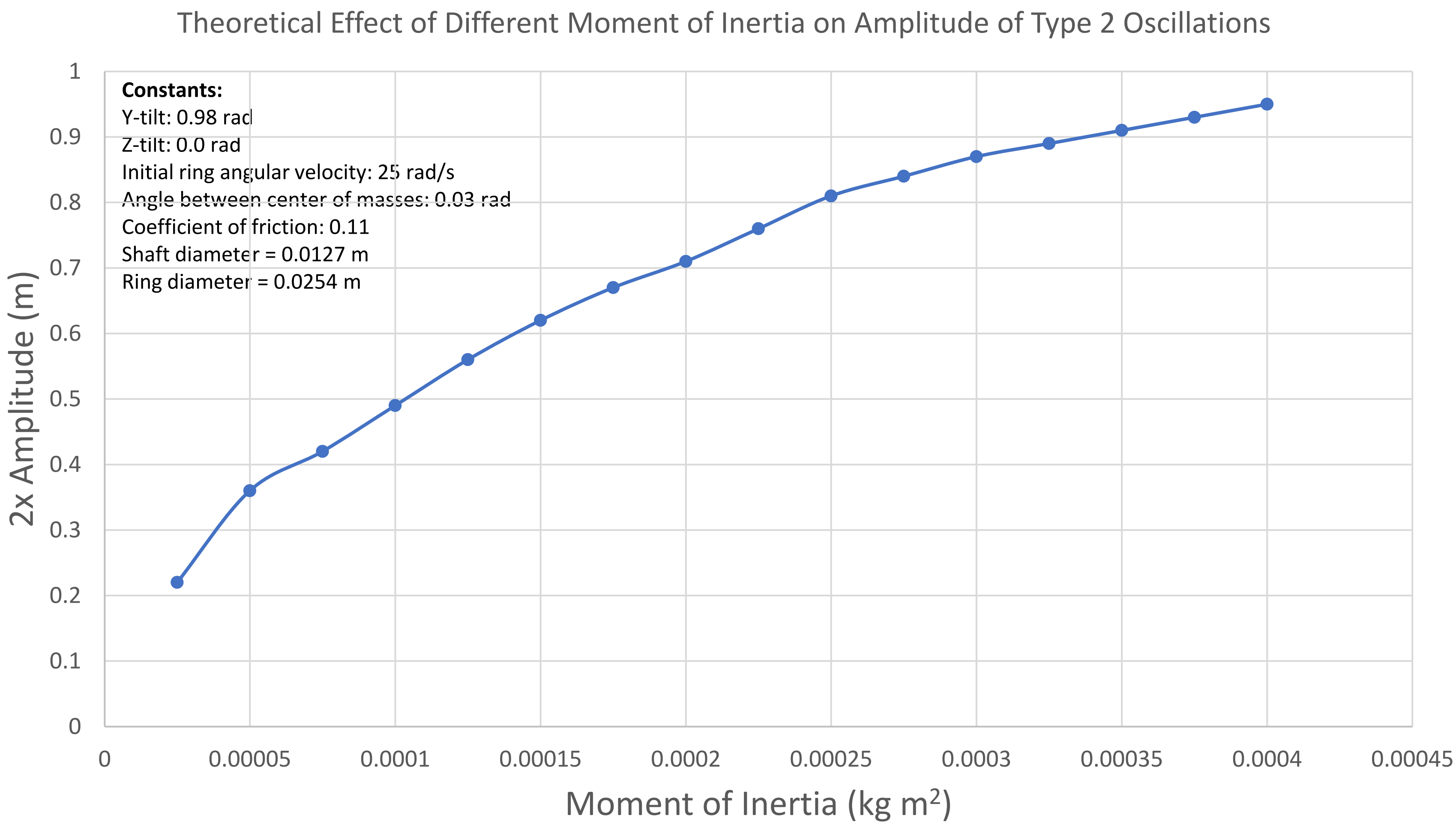
Different shaft angular velocities



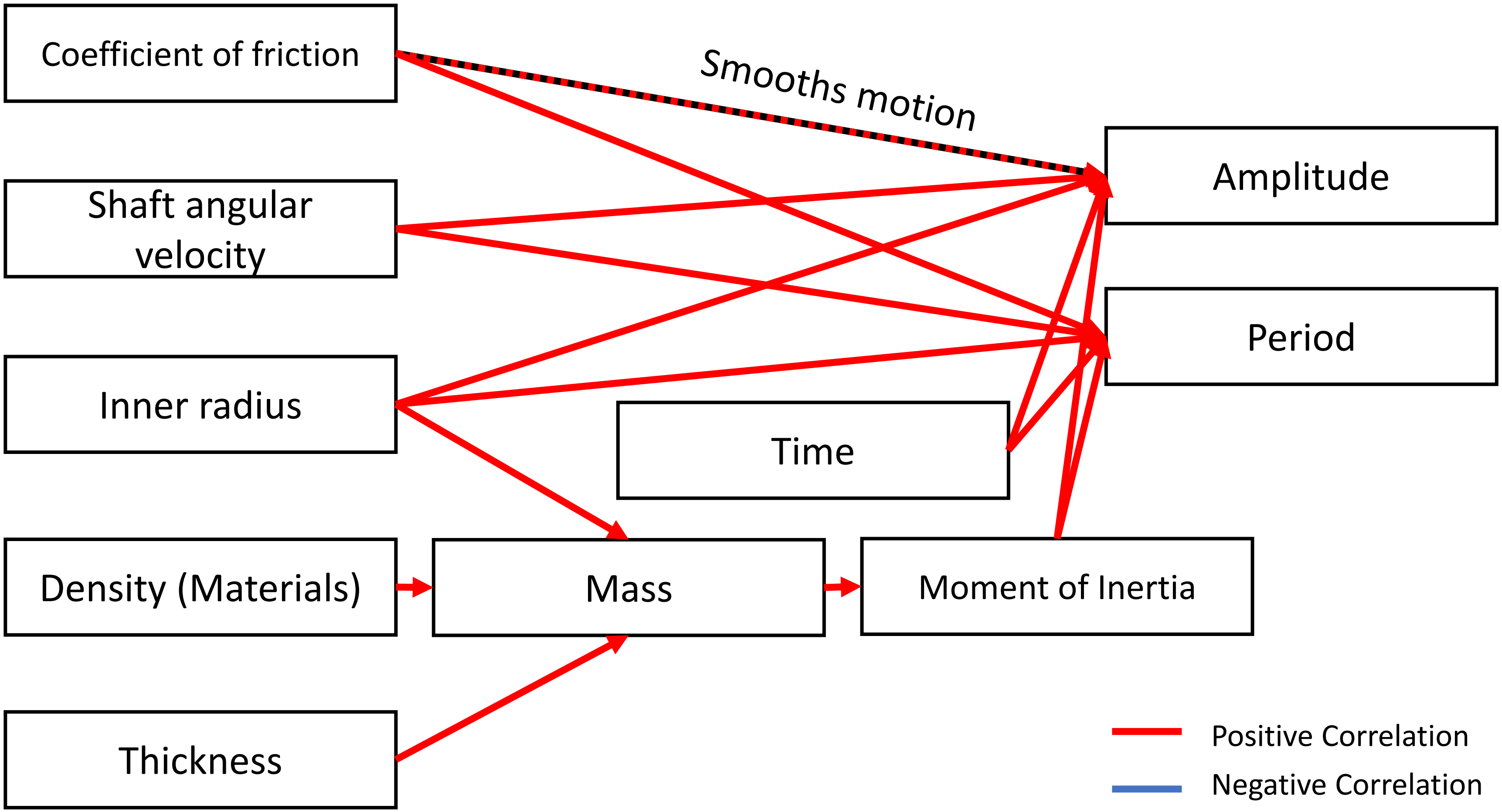
Different moment of inertias



Different moment of inertias



Key Parameters



Conclusion

"An oiled horizontal cylindrical shaft rotates around its axis at constant speed. Make a ring from a cardboard disc with the inner diameter roughly twice the diameter of the shaft and put the ring on the shaft. Depending on the tilt of the ring, it can travel along the shaft in either direction. Investigate the phenomenon."

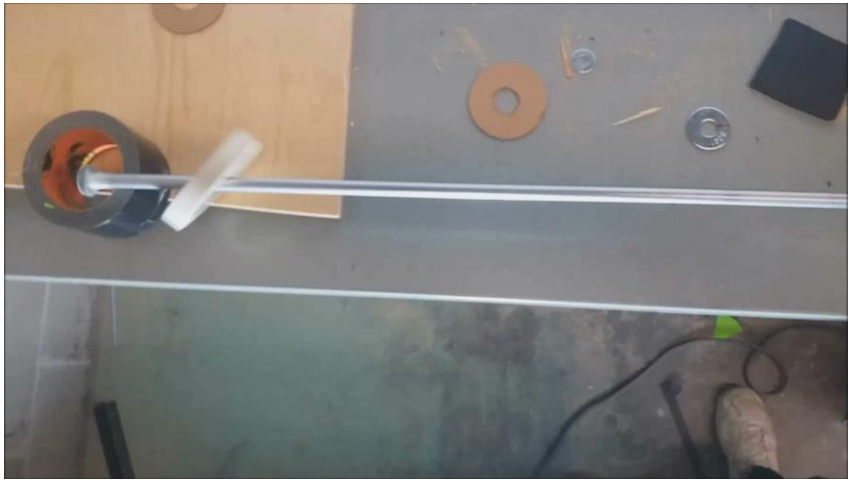
No movement along the shaft	One-directional movement	Oscillatory movement
No tilt	Driving mechanism – Friction force	Two types of oscillations

- *Force balance, torque balance and change in angular momentum*
- *Effect of different tilts on the motion*
- *Amplitude and period of motion*
- *Coefficient of friction, inner radius, outer radius, and shaft angular velocity*

Thank you for listening

Appendix B

Movement of a masking tape roll with significant thickness along a shaft spinning at a constant angular velocity



Appendix C - Moment of inertia tensor

Due to three tilts in all three axis, the moment of inertia is defined as the following tensor:

$$\begin{matrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{matrix}$$

$$I_{xx} = \int (y^2 + z^2) dm \qquad I_{xy} = I_{yx} = -\int xy\, dm$$

$$I_{yy} = \int (x^2 + z^2) dm \qquad I_{yz} = I_{zy} = -\int yz\, dm$$

$$I_{zz} = \int (x^2 + y^2) dm \qquad I_{xz} = I_{zx} = -\int xz\, dm$$

Appendix D - Matlab simulation code

```
clear all
% positive x is along shaft;
% positive y is parallel and opposite to gravity;
% positive z is out of the page

% approach: forces and then kinematics approach to linear dynamics
% torque and change in angular momentum appraoach to rotational dynamics

% GIVEN CONSTANTS
C_RING_MASS = 0.020;
C_GRAVITY = -9.81;
C_STEPS = 51;
C_TOTAL_TIME = 1.67;
C_ORIGIN = [0 0 0];
C_SHAFT_ANGULAR_VELOCITY = 150;
C_FRICTION_COEFFICIENT = 0.1;
C_INNER_RING_RADIUS = 2.54/100;
C_SHAFT_RADIUS = 1.27/100;

% CALCULATED CONSTANTS
dt = C_TOTAL_TIME/(C_STEPS - 1);
% shaft angular velocity vector is strictly restricted to the x-direction
shaftAngularVelocity = [C_SHAFT_ANGULAR_VELOCITY 0 0];

% INITIAL VALUES
% first tilt is zero; second tilt is from above; leave third tilt as zero
%     reference frame: looking from positive y downwards; positive angle
%     is clockwise from negative z axis, negative angle is
%     counter-clockwise from negative z axis
I_TILT = [0 0.979 0];
```

Appendix D - Matlab simulation code

```
% initial magnitude of ring angular velocity
I_RING_ANGULAR_VELOCITY = 75;
% calculate components of ring angular velocity based on magnitude
I_RING_ANGULAR_VELOCITY = [I_RING_ANGULAR_VELOCITY*cos(I_TILT(2)) 0 I_RING_ANGULAR_VELOCITY*sin(I_TILT(2))];
% initial position vector
I_POSITION = [0 0 0];
% initial angle between the center of mass of the ring and the shaft when
% looking straight down the shaft from the negative to the positive x
% direction
I_COM_ANGLE = 0.03;

COMAngle = I_COM_ANGLE;
% linear position
l_position = [0 -(C_INNER_RING_RADIUS-(C_SHAFT_RADIUS/2))*cos(COMAngle) -(C_INNER_RING_RADIUS-
(C_SHAFT_RADIUS/2))*sin(COMAngle)];
l_position = l_position + I_POSITION;
l_velocity = [0 0 0];
l_acceleration1 = [0 0 0];
r_angularVelocity = I_RING_ANGULAR_VELOCITY;
l_outerShaftVelocity = cross(shaftAngularVelocity, [0 cos(COMAngle)*C_SHAFT_RADIUS sin(COMAngle)*C_SHAFT_RADIUS]);
% assign y as 1, find other components relative to 1, to get vector
% direction
u_directionFromCOMToContactPoint = [-tan(COMAngle)*(r_angularVelocity(1)/r_angularVelocity(3)) 1 tan(COMAngle)];
% find vector with magnitude of inner ring radius, and direction of vector
% calculated right before this one
u_COMToContactPoint =
[(C_INNER_RING_RADIUS/norm(u_directionFromCOMToContactPoint))*u_directionFromCOMToContactPoint(1)
(C_INNER_RING_RADIUS/norm(u_directionFromCOMToContactPoint))*u_directionFromCOMToContactPoint(2)
(C_INNER_RING_RADIUS/norm(u_directionFromCOMToContactPoint))*u_directionFromCOMToContactPoint(3)];
l_tangentialVelocity = cross(r_angularVelocity, u_COMToContactPoint) + l_velocity;
l_relativeLinearVelocity = l_outerShaftVelocity - l_tangentialVelocity;
```

Appendix D - Matlab simulation code

```
% forces
f_gravity = [0 C_GRAVITY*C_RING_MASS 0];
f_normal = [0 -C_GRAVITY*C_RING_MASS -C_GRAVITY*C_RING_MASS*tan(COMAngle)];
f_friction = [((norm(f_normal)*C_FRICTION_COEFFICIENT)/norm(l_relativeLinearVelocity))*l_relativeLinearVelocity(1)
((norm(f_normal)*C_FRICTION_COEFFICIENT)/norm(l_relativeLinearVelocity))*l_relativeLinearVelocity(2)
((norm(f_normal)*C_FRICTION_COEFFICIENT)/norm(l_relativeLinearVelocity))*l_relativeLinearVelocity(3)];
f_net = f_gravity + f_normal + f_friction;

t_normal = cross(u_COMToContactPoint, f_normal);
t_friction = cross(u_COMToContactPoint, f_friction);
t_net = t_normal + t_friction;

% center of mass
l_acceleration2 = l_acceleration1 + f_net/C_RING_MASS;
l_velocity = (0.5*(l_acceleration1 + l_acceleration2))*dt;
px(1) = l_position (1);
py(1) = l_position(2);
pz(1) = l_position(3);
l_position = l_position + (l_velocity * dt) + (0.5*(0.5*(l_acceleration1 + l_acceleration2))*dt*dt);

t = 0;

% angular momentum
r_MOI = 0.5;
r_angularMomentum = r_MOI * r_angularVelocity;
t_net_temp = [t_net(1) t_net(2) t_net(3)];
r_angularMomentum = r_angularMomentum + (t_net_temp*dt);
r_angularVelocity = r_angularMomentum / r_MOI;

rx(1) = 0;
```

Appendix D - Matlab simulation code

```
for i = 1:C_STEPS - 1
    t(i+1) = t(i) + dt;

    f_gravity = [0 C_GRAVITY*C_RING_MASS 0];
    f_normal = [0 -C_GRAVITY*C_RING_MASS -C_GRAVITY*C_RING_MASS*tan(COMAngle)];
    f_friction = [((norm(f_normal)*C_FRICTION_COEFFICIENT)/norm(l_relativeLinearVelocity))*l_relativeLinearVelocity(1)
((norm(f_normal)*C_FRICTION_COEFFICIENT)/norm(l_relativeLinearVelocity))*l_relativeLinearVelocity(2)
((norm(f_normal)*C_FRICTION_COEFFICIENT)/norm(l_relativeLinearVelocity))*l_relativeLinearVelocity(3)];
    f_net = f_gravity + f_normal + f_friction;

    t_normal = cross(u_COMToContactPoint, f_normal);
    t_friction = cross(u_COMToContactPoint, f_friction);
    t_net = t_normal + t_friction;

    l_acceleration1 = l_acceleration2;

    l_acceleration2 = l_acceleration1 + f_net/C_RING_MASS;
    l_velocity = (0.5*(l_acceleration1 + l_acceleration2))*dt;
    px(i+1) = l_position(1);
    py(i+1) = l_position(2);
    pz(i+1) = l_position(3);
    l_position = l_position + (l_velocity * dt) + (0.5*(0.5*(l_acceleration1 + l_acceleration2))*dt*dt);

    r_angularMomentum = r_angularMomentum + (t_net_temp*dt);
    r_angularVelocity = r_angularMomentum / r_MOI;

    rx(i+1) = r_angularVelocity(2);
```

Appendix D - Matlab simulation code

```
    COMAngle = atan(l_position(2)/l_position(3));
    l_outerShaftVelocity = cross(shaftAngularVelocity, [0 cos(COMAngle)*C_SHAFT_RADIUS sin(COMAngle)*C_SHAFT_RADIUS]);
    % assign y as 1, find other components relative to 1, to get vector
    % direction
    u_directionFromCOMToContactPoint = [-tan(COMAngle)*(r_angularVelocity(1)/r_angularVelocity(3)) 1 tan(COMAngle)];
    % find vector with magnitude of inner ring radius, and direction of vector
    % calculated right before this one
    u_COMToContactPoint =
[(C_INNER_RING_RADIUS/norm(u_directionFromCOMToContactPoint))*u_directionFromCOMToContactPoint(1)
(C_INNER_RING_RADIUS/norm(u_directionFromCOMToContactPoint))*u_directionFromCOMToContactPoint(2)
(C_INNER_RING_RADIUS/norm(u_directionFromCOMToContactPoint))*u_directionFromCOMToContactPoint(3)];
    l_tangentialVelocity = cross(r_angularVelocity, u_COMToContactPoint) + l_velocity;
    l_relativeLinearVelocity = l_outerShaftVelocity - l_tangentialVelocity;
end
figure(1);
plot(t, px); hold on;
title('Effect of different angular velocities on x-position vs. Time');
xlabel('Time (s)');
ylabel('X-Position (m)');
legend('50 rad/s','100 rad/s','150 rad/s','Location','northwest')
```

References

Concluding Remarks

"An oiled horizontal cylindrical shaft rotates around its axis at constant speed. Make a ring from a cardboard disc with the inner diameter roughly twice the diameter of the shaft and put the ring on the shaft. Depending on the tilt of the ring, it can travel along the shaft in either direction. Investigate the phenomenon."

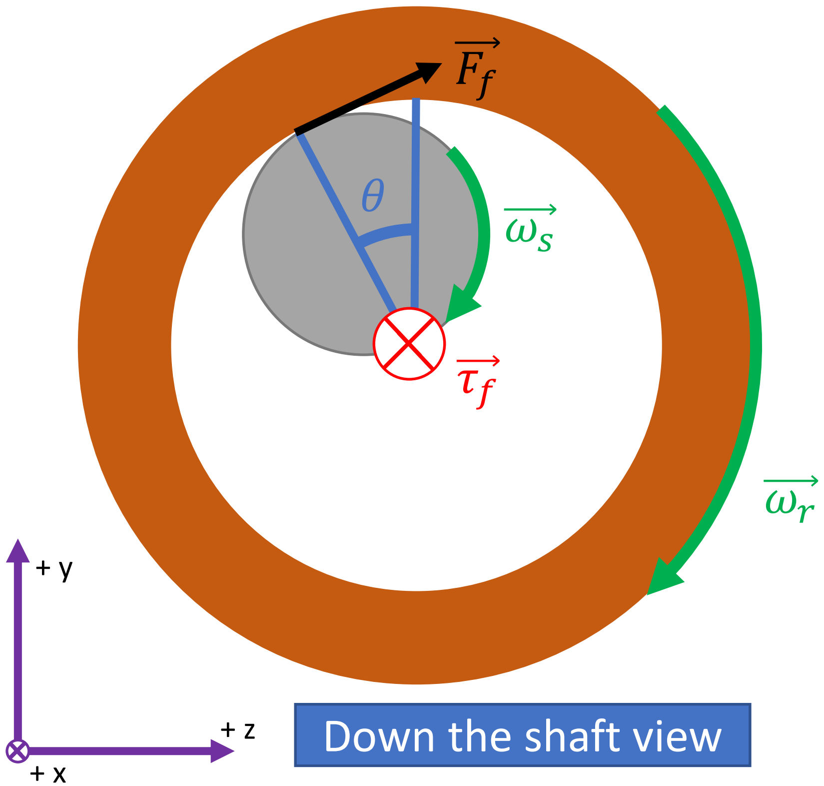
Response to the Opponent

1. *Opponent's Point 1*
2. *Opponent's Point 2*
3. *Opponent's Point 3*

Response to the Reviewer

1. *Opponent's Point 1*
2. *Opponent's Point 2*

Torque Balance



With respect to the center of mass of the ring, only the friction force has a torque:

$$\overrightarrow{\tau_{net}} = \overrightarrow{\tau_f}$$

$$\overrightarrow{\tau_{net}} = \overrightarrow{r_r} \times \overrightarrow{F_f}$$

Initially:

$$\vec{L} = 0$$

After ring is placed on shaft:

$$\frac{d\vec{L}}{dt} = \overrightarrow{\tau_{net}}$$